

ANGLES, PARALLEL LINES, AND POLYGONS FOR AMC 8

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1. ANGLES

This section covers how angles are measured and the pairs of angles that appear when lines cross. Most problems in this handout are solved by **angle chasing**: starting from the given angles, the facts below give new angles one at a time until the required angle is reached.

1.1. Measuring Angles. An **angle** is formed by two rays with a common endpoint, called the **vertex**. The angle with vertex B formed by rays BA and BC is written $\angle ABC$ or $\angle CBA$, with the vertex in the middle. When only one angle at B is under discussion, it can be written $\angle B$. The same symbol also stands for the measure of the angle, so $\angle ABC = 40^\circ$ means that the angle measures 40 degrees. A full turn around a point measures 360° .

Definition 1.1.1. An angle is

- **acute** if it measures more than 0° and less than 90° ;
- a **right angle** if it measures 90° , a quarter of a full turn;
- **obtuse** if it measures more than 90° and less than 180° ;
- a **straight angle** if it measures 180° , half of a full turn. The two rays of a straight angle point in opposite directions and form a line.

Two lines are **perpendicular** if they meet at a right angle. In figures, a right angle is marked with a small square.

Angles can be added and subtracted like lengths.

Proposition 1.1.2 (Angle Addition). If ray BD lies inside $\angle ABC$, then $\angle ABD + \angle DBC = \angle ABC$. In particular, angles that together make up a straight angle add to 180° , and angles around a point that together make up a full turn, without overlapping, add to 360° .

Definition 1.1.3. A ray BD inside $\angle ABC$ **bisects** the angle if $\angle ABD = \angle DBC$. The ray is then the **angle bisector**, and each of the two smaller angles is half of $\angle ABC$.

Examples.

Problem 1.1.4 (1987 AJHSME, Problem #4). Martians measure angles in clerts. There are 500 clerts in a full circle. How many clerts are there in a right angle?

(A) 90 (B) 100 (C) 125 (D) 180 (E) 250

Strategy. A right angle is the same fraction of a full turn in any unit of measure.

Solution. A right angle measures 90° and a full circle measures 360° , so a right angle is

$$\frac{90}{360} = \frac{1}{4}$$

of a full circle. This fraction does not depend on the unit, so a right angle is $\frac{1}{4}$ of 500 clerts, which is 125 clerts.

Answer: (C) 125. □

Problem 1.1.5 (2012 AMC 10A, Problem #4). Let $\angle ABC = 24^\circ$ and $\angle ABD = 20^\circ$. What is the smallest possible degree measure for $\angle CBD$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 12

Strategy. Both angles have ray BA as a side. Ray BD is either on the same side of BA as ray BC or on the opposite side. Use angle addition in each case.

Solution. If ray BD is on the same side of BA as ray BC , then BD lies inside $\angle ABC$, because $20^\circ < 24^\circ$. By angle addition, $\angle ABD + \angle DBC = \angle ABC$, so

$$\angle CBD = 24^\circ - 20^\circ = 4^\circ.$$

If ray BD is on the other side of BA , then ray BA lies inside $\angle CBD$, and

$$\angle CBD = 24^\circ + 20^\circ = 44^\circ.$$

The smallest possible measure is 4° .

Answer: (C) 4. □

1.2. Complementary and Supplementary Angles.

Definition 1.2.1. Two angles are **complementary** if their measures add to 90° , and **supplementary** if their measures add to 180° . Each angle is then the **complement** or the **supplement** of the other. The complement of an angle of x° measures $(90 - x)^\circ$, and its supplement measures $(180 - x)^\circ$.

Two angles that share a side, and whose other sides point in opposite directions, together make a straight angle. Such a pair is called a **linear pair**, and its two angles are supplementary. In the same way, two angles that share a side and together make a right angle are complementary.

A problem that describes angles through their complements or supplements becomes an equation once each complement is written as $90 - x$ and each supplement as $180 - x$.

Example.

Problem 1.2.2 (2016 AMC 10B, Problem #7). The ratio of the measures of two acute angles is $5 : 4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

(A) 75 (B) 90 (C) 135 (D) 150 (E) 270

Strategy. Write the angles as $5x^\circ$ and $4x^\circ$. The larger angle has the smaller complement, so the complement of the smaller angle is the one that is twice as large.

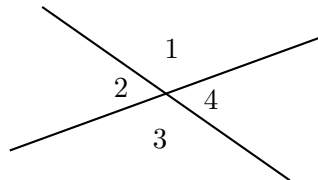
Solution. Let the angles measure $5x^\circ$ and $4x^\circ$. Their complements measure $(90 - 5x)^\circ$ and $(90 - 4x)^\circ$. Since $5x > 4x$, the first complement is the smaller one, so

$$90 - 4x = 2(90 - 5x) = 180 - 10x.$$

Then $6x = 90$, so $x = 15$. The angles measure 75° and 60° , their complements measure 15° and 30° , and $30 = 2 \cdot 15$ as required. The sum of the two angles is $75 + 60 = 135$.

Answer: (C) 135. □

1.3. Vertical Angles. When two lines cross, they form four angles. Two of these angles that are across from each other, such as $\angle 1$ and $\angle 3$ in the figure, are called **vertical angles**.



Theorem 1.3.1 (Vertical Angles). *Vertical angles are equal: in the figure, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.*

Proof. The angles $\angle 1$ and $\angle 2$ form a linear pair, and so do $\angle 2$ and $\angle 3$. Hence

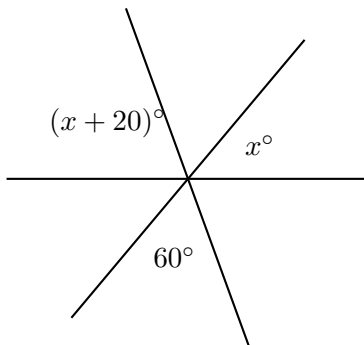
$$\angle 1 + \angle 2 = 180^\circ = \angle 2 + \angle 3,$$

which gives $\angle 1 = \angle 3$. The same argument with $\angle 3$ and $\angle 4$ gives $\angle 2 = \angle 4$. □

So when one of the four angles is known, all four are known: two of them equal the known angle, and the other two equal its supplement.

Example.

Problem 1.3.2. In the figure, three lines meet at one point. Find x .



Strategy. The three marked angles are not next to each other. The unmarked angle between x° and $(x + 20)^\circ$ is vertical to the 60° angle.

Solution. Above the horizontal line there are three angles: x° , $(x + 20)^\circ$, and the unmarked angle between them. The unmarked angle and the 60° angle are vertical angles, so the unmarked angle also measures 60° . The three angles above the horizontal line make up a straight angle, so

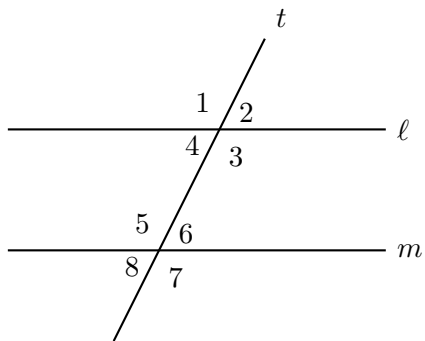
$$x + 60 + (x + 20) = 180.$$

Then $2x = 100$, so $x = 50$.

Answer: 50. □

2. PARALLEL LINES

Two lines in a plane are **parallel** if they never meet. The notation $\ell \parallel m$ means that lines ℓ and m are parallel. A **transversal** is a line that crosses two other lines. A transversal t that crosses lines ℓ and m forms eight angles, four at each crossing point, numbered as in the figure.



Definition 2.1. In the figure,

- **corresponding angles** are in matching positions at the two crossings: $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$;
- **alternate interior angles** lie between ℓ and m on opposite sides of t : $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$;
- **same-side interior angles** lie between ℓ and m on the same side of t : $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$.

Theorem 2.2 (Parallel Lines). *If $\ell \parallel m$, then*

- (a) *corresponding angles are equal;*
- (b) *alternate interior angles are equal;*
- (c) *same-side interior angles are supplementary.*

Proof. Statement (a) is a basic property of parallel lines, taken here without proof. For (b), $\angle 4 = \angle 2$ because they are vertical angles, and $\angle 2 = \angle 6$ by (a), so $\angle 4 = \angle 6$. The same argument gives $\angle 3 = \angle 5$. For (c), $\angle 5$ and $\angle 6$ form a linear pair, so

$$\angle 4 + \angle 5 = \angle 6 + \angle 5 = 180^\circ,$$

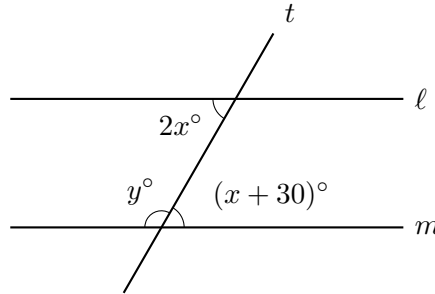
and in the same way $\angle 3 + \angle 6 = 180^\circ$. □

When $\ell \parallel m$ and t is not perpendicular to them, the eight angles therefore take only two values: the four acute angles are equal, the four obtuse angles are equal, and each acute angle is supplementary to each obtuse angle.

When a point lies between two parallel lines, it often helps to draw an extra line through that point, parallel to the other two. This creates new pairs of alternate interior angles, as in the second example below.

Examples.

Problem 2.3. In the figure, $\ell \parallel m$. Find x and y .



Strategy. Decide how each pair of marked angles is placed relative to ℓ , m and t : as alternate interior angles or as same-side interior angles.

Solution. The angles $2x^\circ$ and $(x + 30)^\circ$ lie between ℓ and m on opposite sides of t , so they are alternate interior angles and are equal:

$$2x = x + 30,$$

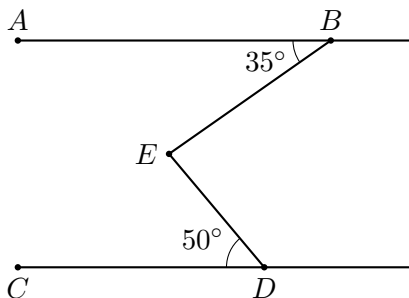
which gives $x = 30$. Both angles measure 60° . The angles $2x^\circ$ and y° lie between ℓ and m on the same side of t , so they are same-side interior angles and are supplementary:

$$y = 180 - 60 = 120.$$

As a check, y° and $(x + 30)^\circ$ form a linear pair, and $120 + 60 = 180$.

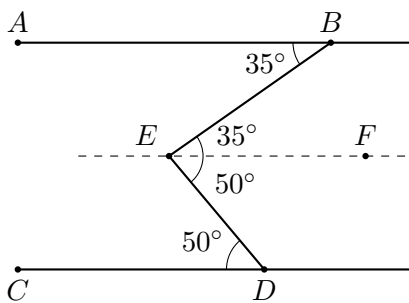
Answer: $x = 30$ and $y = 120$. □

Problem 2.4. In the figure, $AB \parallel CD$, $\angle ABE = 35^\circ$, and $\angle CDE = 50^\circ$. Find $\angle BED$.



Strategy. The given angles are formed by two different transversals, BE and DE . Draw the line through E parallel to AB . It splits $\angle BED$ into two angles, and each of them is an alternate interior angle to one of the given angles.

Solution. Draw the line through E parallel to AB . It is also parallel to CD . Choose a point F on it to the right of E , as shown.



With BE as the transversal of the parallel lines AB and EF , the angles $\angle ABE$ and $\angle BEF$ are alternate interior angles, so $\angle BEF = 35^\circ$. With DE as the transversal of the parallel lines CD and EF , the angles $\angle CDE$ and $\angle DEF$ are alternate interior angles, so $\angle DEF = 50^\circ$. Ray EF lies inside $\angle BED$, so

$$\angle BED = \angle BEF + \angle FED = 35^\circ + 50^\circ = 85^\circ.$$

Answer: $\angle BED = 85^\circ$. □

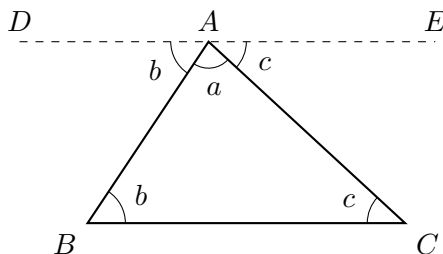
3. TRIANGLES

This section proves the two facts about triangles that are used most often in angle chasing.

3.1. The Triangle Angle Sum.

Theorem 3.1.1 (Triangle Angle Sum). *The angles of a triangle add to 180° . That is, in triangle ABC ,*

$$\angle A + \angle B + \angle C = 180^\circ.$$



Proof. Draw the line through A parallel to BC , and choose points D and E on it on either side of A , with D on the side of B and E on the side of C , as in the figure. With AB as the transversal of the parallel lines DE and BC , the angles $\angle DAB$ and $\angle ABC$ are alternate interior angles, so they are equal. With AC as the transversal, $\angle EAC = \angle ACB$ in the same way. The angles $\angle DAB$, $\angle BAC$ and $\angle CAE$ together make up the straight angle DAE , so

$$\angle B + \angle A + \angle C = \angle DAB + \angle BAC + \angle CAE = 180^\circ. \quad \square$$

So the third angle of a triangle is 180° minus the sum of the other two. Since a right angle measures 90° , the theorem has the following consequence.

Corollary 3.1.2. *In a right triangle, the two acute angles add to 90° , so they are complementary.*

Remark 3.1.3. A triangle with two equal sides is **isosceles**. The angles opposite the two equal sides are equal.

Examples.

Problem 3.1.4 (2017 AMC 8, Problem #6). If the degree measures of the angles of a triangle are in the ratio $3 : 3 : 4$, what is the degree measure of the largest angle of the triangle?

- (A) 18 (B) 36 (C) 60 (D) 72 (E) 90

Strategy. Write the angles as $3k$, $3k$ and $4k$ degrees and use the angle sum.

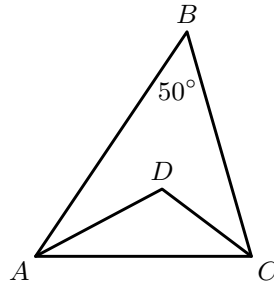
Solution. Let the angles measure $3k^\circ$, $3k^\circ$ and $4k^\circ$. By the triangle angle sum,

$$3k + 3k + 4k = 10k = 180,$$

so $k = 18$. The largest angle measures $4k = 72$ degrees.

Answer: (D) 72. $\square$

Problem 3.1.5 (1996 AJHSME, Problem #24). The measure of angle ABC is 50° . $\overline{AD}$ bisects angle BAC , and $\overline{DC}$ bisects angle BCA . The measure of angle ADC is



- (A) 90° (B) 100° (C) 115° (D) 122.5° (E) 125°

Strategy. The angles $\angle BAC$ and $\angle BCA$ cannot be found separately, but their sum can. Work with the sum.

Solution. In triangle ABC ,

$$\angle BAC + \angle BCA = 180^\circ - 50^\circ = 130^\circ.$$

Since AD and CD bisect these two angles, $\angle DAC = \frac{1}{2}\angle BAC$ and $\angle DCA = \frac{1}{2}\angle BCA$, so

$$\angle DAC + \angle DCA = \frac{1}{2} \cdot 130^\circ = 65^\circ.$$

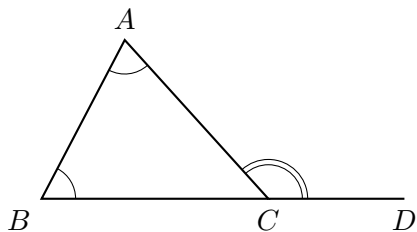
In triangle ADC ,

$$\angle ADC = 180^\circ - 65^\circ = 115^\circ.$$

Answer: (C) 115° . $\square$

3.2. The Exterior Angle Theorem.

Definition 3.2.1. Extend side BC of triangle ABC past C to a point D . The angle $\angle ACD$, between side CA and the extension, is an **exterior angle** of the triangle at C . The angles of the triangle at the other two vertices, $\angle A$ and $\angle B$, are its **remote interior angles**.



Theorem 3.2.2 (Exterior Angle Theorem). *An exterior angle of a triangle equals the sum of its two remote interior angles. In the figure,*

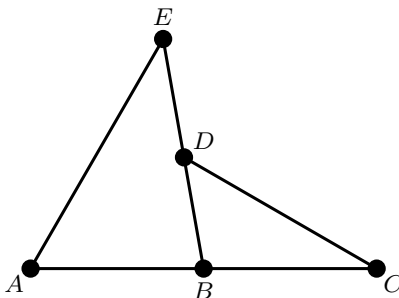
$$\angle ACD = \angle A + \angle B.$$

Proof. The angles $\angle ACB$ and $\angle ACD$ form a linear pair, so $\angle ACD = 180^\circ - \angle ACB$. By the triangle angle sum, $\angle A + \angle B = 180^\circ - \angle ACB$ as well. $\square$

The theorem combines a triangle sum and a supplement into one step. It is most useful when the angle asked for, or an angle that is easy to find, is an exterior angle of some triangle in the figure. The main task is to find that triangle.

Examples.

Problem 3.2.3 (1994 AJHSME, Problem #7). If $\angle A = 60^\circ$, $\angle E = 40^\circ$, and $\angle C = 30^\circ$, then $\angle BDC =$



- (A) 40° (B) 50° (C) 60° (D) 70° (E) 80°

Strategy. The angle $\angle ABE$ is an exterior angle of triangle DBC . Find it from triangle ABE .

Solution. In triangle ABE ,

$$\angle ABE = 180^\circ - 60^\circ - 40^\circ = 80^\circ.$$

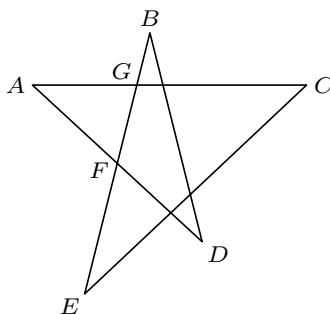
Point D lies on $\overline{BE}$, so $\angle ABD = \angle ABE = 80^\circ$. Since A , B and C lie on a line, $\angle ABD$ is the exterior angle of triangle DBC at B . Its remote interior angles are $\angle BDC$ and $\angle BCD = 30^\circ$, so

$$80^\circ = \angle BDC + 30^\circ,$$

and $\angle BDC = 50^\circ$.

Answer: (B) 50° . $\square$

Problem 3.2.4 (2000 AMC 8, Problem #24). If $\angle A = 20^\circ$ and $\angle AFG = \angle AGF$, then $\angle B + \angle D =$



- (A) 48° (B) 60° (C) 72° (D) 80° (E) 90°

Strategy. The angles $\angle B$ and $\angle D$ are two angles of triangle BFD . Look for an exterior angle of that triangle that is easy to find.

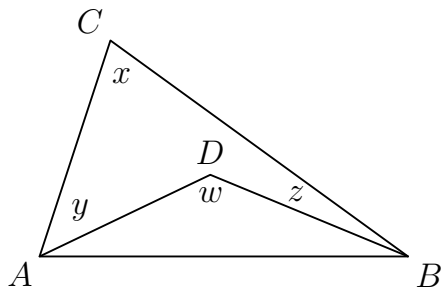
Solution. In triangle AFG , $\angle AFG + \angle AGF = 180^\circ - 20^\circ = 160^\circ$, and the two angles are equal, so $\angle AFG = 80^\circ$.

Point G lies on $\overline{FB}$, so $\angle AFG$ is the same angle as $\angle AFB$. Point F lies on $\overline{AD}$, so $\angle AFB$ is the exterior angle of triangle BFD at F , and its remote interior angles are $\angle B$ and $\angle D$. By the exterior angle theorem,

$$\angle B + \angle D = \angle AFB = 80^\circ.$$

Answer: (D) 80° . □

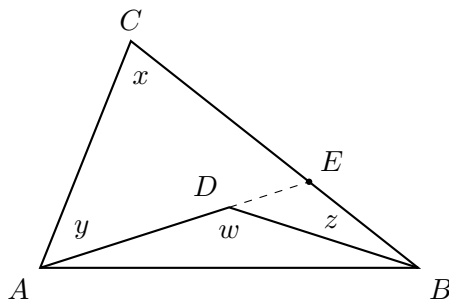
Problem 3.2.5 (1987 AHSME, Problem #6). In the $\triangle ABC$ shown, D is some interior point, and x, y, z, w are the measures of angles in degrees. Solve for x in terms of y, z , and w .



- (A) $w - y - z$ (B) $w - 2y - 2z$ (C) $180 - w - y - z$ (D) $2w - y - z$
 (E) $180 - w + y + z$

Strategy. Extend $\overline{AD}$ until it meets $\overline{BC}$. This makes two triangles, each with a useful exterior angle.

Solution. Extend $\overline{AD}$ past D to meet $\overline{BC}$ at E .



In triangle ACE , the angle $\angle AEB$ is the exterior angle at E , because B lies on the extension of $\overline{CE}$ past E . Its remote interior angles are $\angle ACE = x^\circ$ and $\angle CAE = y^\circ$, so $\angle AEB = (x + y)^\circ$.

In triangle DEB , the angle $\angle ADB$ is the exterior angle at D , because A lies on the extension of $\overline{ED}$ past D . Its remote interior angles are $\angle DEB = (x + y)^\circ$ and $\angle DBE = z^\circ$. Therefore

$$w = (x + y) + z,$$

and $x = w - y - z$.

Answer: (A) $w - y - z$. □

Remark 3.2.6. The same argument works for every point D inside triangle ABC , so in general

$$\angle ADB = \angle ACB + \angle CAD + \angle CBD.$$

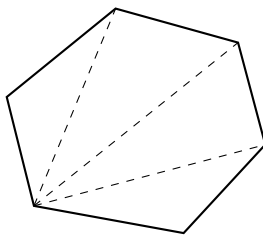
4. POLYGONS

A **polygon** is a closed figure made of segments, called **sides**, that meet only at their endpoints, called **vertices**. A polygon with n sides is an **n -gon**. For example, a 5-gon is a pentagon and a 6-gon is a hexagon. A **diagonal** is a segment that joins two vertices that are not next to each other. A polygon is **convex** if each of its interior angles is less than 180° . All polygons in this section are convex.

4.1. The Interior Angle Sum.

Theorem 4.1.1 (Interior Angle Sum). *The interior angles of a convex n -gon add to $(n - 2) \cdot 180^\circ$.*

Proof. Choose one vertex and draw the diagonals from it to the $n - 3$ vertices that are not next to it. These diagonals divide the polygon into $n - 2$ triangles. The figure shows this for $n = 6$.



Together, the angles of these triangles make up exactly the interior angles of the polygon. Each triangle contributes 180° , so the interior angles add to $(n - 2) \cdot 180^\circ$. □

For example, the interior angles of a quadrilateral add to 360° , those of a pentagon add to 540° , and those of a hexagon add to 720° . Each time a side is added, the sum grows by 180° .

Examples.

Problem 4.1.2 (2002 AMC 12B, Problem #5). Let v , w , x , y , and z be the degree measures of the five angles of a pentagon. Suppose $v < w < x < y < z$ and v , w , x , y , z form an arithmetic sequence. Find the value of x .

(A) 72 (B) 84 (C) 90 (D) 108 (E) 120

Strategy. In an arithmetic sequence, consecutive terms differ by the same amount d . Write all five terms in terms of the middle term x and d .

Solution. The interior angles of a pentagon add to $(5 - 2) \cdot 180 = 540$ degrees. With common difference d , the five angles are

$$v = x - 2d, \quad w = x - d, \quad x, \quad y = x + d, \quad z = x + 2d.$$

Their sum is $5x$, because the terms with d cancel. So $5x = 540$ and $x = 108$.

Answer: (D) 108. □

Problem 4.1.3 (2017 AMC 12A, Problem #11). Claire adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. She then discovers that she forgot to include one angle. What is the degree measure of the forgotten angle?

(A) 37 (B) 63 (C) 117 (D) 143 (E) 163

Strategy. The true sum is a multiple of 180. The forgotten angle is less than 180° because the polygon is convex.

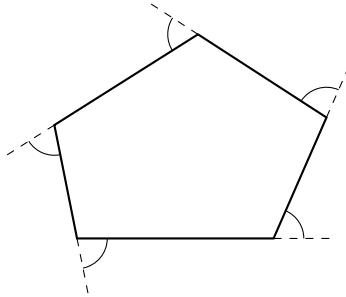
Solution. Let the forgotten angle measure a° , where $0 < a < 180$. The true sum $2017 + a$ is $(n - 2) \cdot 180$ for some n , so it is a multiple of 180 strictly between 2017 and $2017 + 180 = 2197$. Since

$$180 \cdot 11 = 1980 < 2017 \quad \text{and} \quad 180 \cdot 12 = 2160 < 2197 < 180 \cdot 13,$$

the only possibility is 2160. Hence $a = 2160 - 2017 = 143$.

Answer: (D) 143. □

4.2. Exterior Angles. At a vertex V of a convex polygon, extend one of the two sides at V past V . As for a triangle, the angle between this extension and the other side at V is an exterior angle of the polygon at V . The interior angle and the exterior angle at a vertex form a linear pair, so the exterior angle is the supplement of the interior angle. The figure marks one exterior angle at each vertex of a pentagon.



Theorem 4.2.1 (Exterior Angle Sum). *The exterior angles of a convex polygon, one at each vertex, add to 360° .*

Proof. At each of the n vertices, the interior angle and the exterior angle add to 180° , so all $2n$ of these angles add to $n \cdot 180^\circ$. The interior angles add to $(n - 2) \cdot 180^\circ$, so the exterior angles add to

$$n \cdot 180^\circ - (n - 2) \cdot 180^\circ = 360^\circ. \quad \square$$

The exterior angle sum does not depend on n . When the number of sides is unknown, an equation in the exterior angles is often simpler than one in the interior angles.

Example.

Problem 4.2.2. One interior angle of a convex polygon measures 120° , and each of the other interior angles measures 150° . How many sides does the polygon have?

Strategy. Find the exterior angles, which add to 360° whatever the number of sides.

Solution. The exterior angle at the 120° vertex measures $180^\circ - 120^\circ = 60^\circ$, and the exterior angle at each other vertex measures $180^\circ - 150^\circ = 30^\circ$. If the polygon has n sides, there are $n - 1$ exterior angles of 30° , so

$$60 + 30(n - 1) = 360.$$

Then $30(n - 1) = 300$, so $n - 1 = 10$ and $n = 11$. As a check, the interior angles add to $120^\circ + 10 \cdot 150^\circ = 1620^\circ = (11 - 2) \cdot 180^\circ$.

Answer: 11. □

4.3. Regular Polygons. A polygon is **regular** if all of its sides are equal and all of its angles are equal. The n equal angles of a regular n -gon share the total $(n - 2) \cdot 180^\circ$, and its n equal exterior angles share the total 360° . This gives the following.

Corollary 4.3.1. *Each exterior angle of a regular n -gon measures $\frac{360^\circ}{n}$, and each interior angle measures*

$$\frac{(n - 2) \cdot 180^\circ}{n} = 180^\circ - \frac{360^\circ}{n}.$$

To find the number of sides from the size of an interior angle, find the exterior angle first and divide 360° by it.

n	Name	Sum of the angles	Regular n -gon	
			Each angle	Each exterior angle
4	quadrilateral	360°	90°	90°
5	pentagon	540°	108°	72°
6	hexagon	720°	120°	60°
8	octagon	1080°	135°	45°
9	nonagon	1260°	140°	40°
10	decagon	1440°	144°	36°
12	dodecagon	1800°	150°	30°

Examples.

Problem 4.3.2. Each interior angle of a regular polygon measures 156° . How many sides does the polygon have?

Strategy. Find the exterior angle, which is $\frac{360^\circ}{n}$.

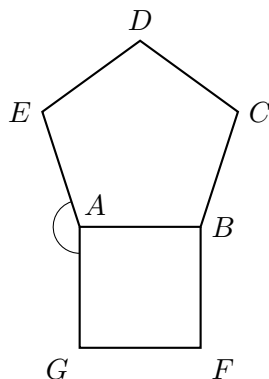
Solution. Each exterior angle measures $180^\circ - 156^\circ = 24^\circ$, so

$$\frac{360}{n} = 24 \quad \text{and} \quad n = \frac{360}{24} = 15.$$

As a check, a regular 15-gon has angles of $\frac{13 \cdot 180^\circ}{15} = 156^\circ$.

Answer: 15. □

Problem 4.3.3. In the figure, $ABCDE$ is a regular pentagon and $ABFG$ is a square. Find $\angle EAG$.



Strategy. Three angles fill the full turn around A : an angle of the pentagon, an angle of the square, and $\angle EAG$.

Solution. Each angle of a regular pentagon measures 108° , so $\angle EAB = 108^\circ$, and each angle of a square measures 90° , so $\angle BAG = 90^\circ$. These two angles and $\angle EAG$ fill the full turn around A without overlapping, so

$$\angle EAG = 360^\circ - 108^\circ - 90^\circ = 162^\circ.$$

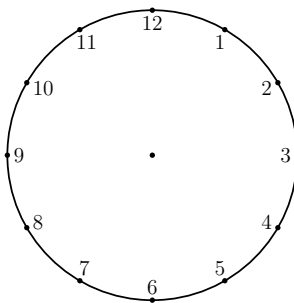
Answer: $\angle EAG = 162^\circ$.

□

5. PROBLEMS

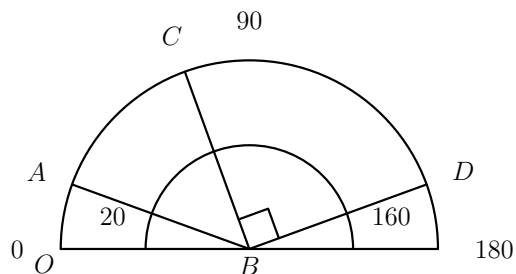
Beginner.

1. (1999 AMC 8, Problem #2) What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?

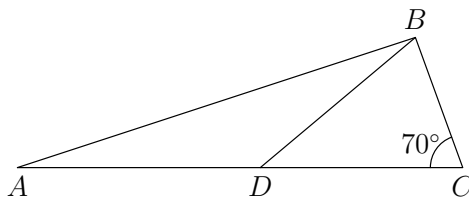


- (A) 30 (B) 45 (C) 60 (D) 75 (E) 90

2. (1988 AJHSME, Problem #5) If $\angle CBD$ is a right angle, then this protractor indicates that the measure of $\angle ABC$ is approximately



- (A) 20° (B) 40° (C) 50° (D) 70° (E) 120°
3. (2020 AMC 10B, Problem #4) The acute angles of a right triangle are a° and b° , where $a > b$ and both a and b are prime numbers. What is the least possible value of b ?
- (A) 2 (B) 3 (C) 5 (D) 7 (E) 11
4. (2014 AMC 8, Problem #9) In $\triangle ABC$, D is a point on side $\overline{AC}$ such that $BD = DC$ and $\angle BCD$ measures 70° . What is the degree measure of $\angle ADB$?



- (A) 100 (B) 120 (C) 135 (D) 140 (E) 150

Intermediate.

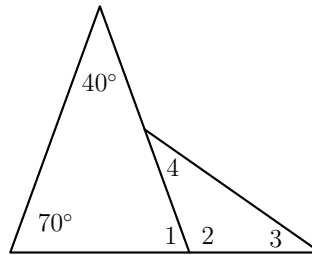
5. (2002 AMC 12A, Problem #4) Find the degree measure of an angle whose complement is 25% of its supplement.
- (A) 48 (B) 60 (C) 75 (D) 120 (E) 150

6. (1997 AJHSME, Problem #12)

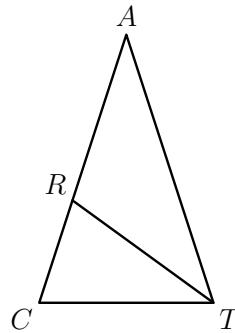
$$\angle 1 + \angle 2 = 180^\circ$$

$$\angle 3 = \angle 4$$

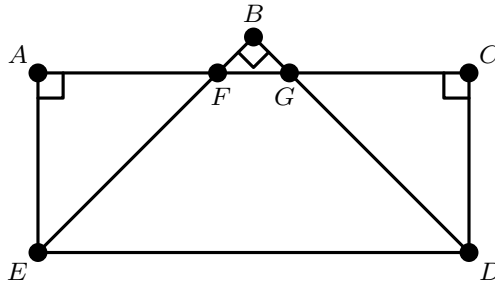
Find $\angle 4$



- (A) 20° (B) 25° (C) 30° (D) 35° (E) 40°
7. (2000 AMC 8, Problem #13) In triangle CAT , we have $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$. If $\overline{TR}$ bisects $\angle ATC$, then $\angle CRT =$



- (A) 16° (B) 51° (C) 72° (D) 90° (E) 108°
8. (1995 AJHSME, Problem #13) In the figure, $\angle A$, $\angle B$ and $\angle C$ are right angles. If $\angle AEB = 40^\circ$ and $\angle BED = \angle BDE$, then $\angle CDE =$

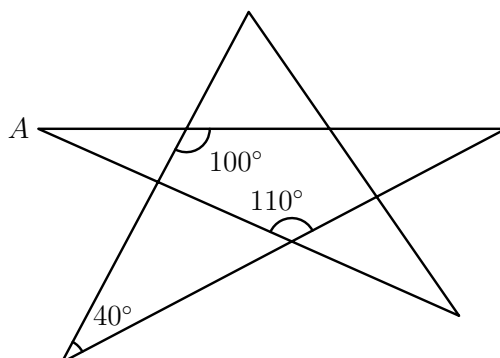


- (A) 75° (B) 80° (C) 85° (D) 90° (E) 95°

Advanced.

9. (1999 AHSME, Problem #7) What is the largest number of acute angles that a convex hexagon can have?
 (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
10. (2007 AMC 10B, Problem #15) The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?
 (A) 125 (B) 144 (C) 153 (D) 173 (E) 180

11. (1990 AHSME, Problem #3) The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is 75° , then the largest angle is
(A) 95° (B) 100° (C) 105° (D) 110° (E) 115°
12. (1999 AMC 8, Problem #21) The degree measure of angle A is



- (A) 20 (B) 30 (C) 35 (D) 40 (E) 45