

ANGLES, PARALLEL LINES, AND POLYGONS FOR AMC 8: SOLUTIONS

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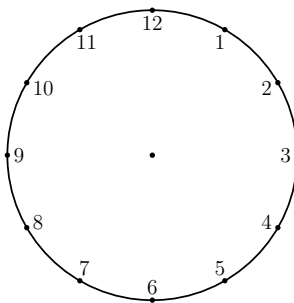
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1. SOLUTIONS

Beginner.

1. (1999 AMC 8, Problem #2) What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?



- (A) 30 (B) 45 (C) 60 (D) 75 (E) 90

Strategy. The twelve hour marks divide a full turn into twelve equal angles.

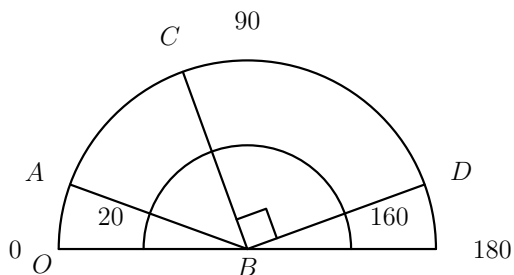
Solution. The twelve hour marks are equally spaced around the clock, so the angle between two neighboring marks is

$$\frac{360^\circ}{12} = 30^\circ.$$

At 10 o'clock the minute hand points at 12 and the hour hand points at 10. From 10 to 12 there are two of these 30° angles, so the smaller angle between the hands is $2 \cdot 30^\circ = 60^\circ$.

Answer: (C) 60. □

2. (1988 AJHSME, Problem #5) If $\angle CBD$ is a right angle, then this protractor indicates that the measure of $\angle ABC$ is approximately



- (A) 20° (B) 40° (C) 50° (D) 70° (E) 120°

Strategy. The protractor gives the angle that each ray makes with ray BO . Use the right angle to find the reading for ray BC .

Solution. The protractor shows that $\angle OBA = 20^\circ$ and $\angle OBD = 160^\circ$. Ray BC lies inside $\angle OBD$ and $\angle CBD = 90^\circ$, so by angle addition

$$\angle OBC = \angle OBD - \angle CBD = 160^\circ - 90^\circ = 70^\circ.$$

Ray BA lies inside $\angle OBC$, so

$$\angle ABC = \angle OBC - \angle OBA = 70^\circ - 20^\circ = 50^\circ.$$

Answer: (C) 50° . □

3. (2020 AMC 10B, Problem #4) The acute angles of a right triangle are a° and b° , where $a > b$ and both a and b are prime numbers. What is the least possible value of b ?

- (A) 2 (B) 3 (C) 5 (D) 7 (E) 11

Strategy. The acute angles of a right triangle are complementary. Try the smallest primes for b .

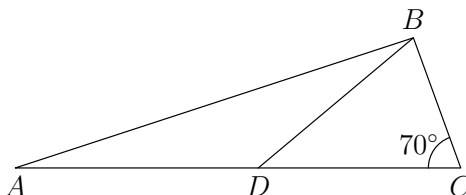
Solution. The acute angles of a right triangle add to 90° , so $a = 90 - b$. Both b and $90 - b$ must be prime. Trying the primes in increasing order:

- $b = 2$ gives $a = 88$, which is even;
- $b = 3$ gives $a = 87 = 3 \cdot 29$;
- $b = 5$ gives $a = 85 = 5 \cdot 17$;
- $b = 7$ gives $a = 83$, which is prime.

Since $83 > 7$, the condition $a > b$ also holds. The least possible value of b is 7.

Answer: (D) 7. □

4. (2014 AMC 8, Problem #9) In $\triangle ABC$, D is a point on side $\overline{AC}$ such that $BD = DC$ and $\angle BCD$ measures 70° . What is the degree measure of $\angle ADB$?



- (A) 100 (B) 120 (C) 135 (D) 140 (E) 150

Strategy. The angle $\angle ADB$ is an exterior angle of triangle BDC .

Solution. In triangle BDC , the sides BD and DC are equal. In a triangle, the angles opposite two equal sides are equal, so

$$\angle DBC = \angle DCB = 70^\circ.$$

Points A , D and C lie on a line, so $\angle ADB$ is the exterior angle of triangle BDC at D . By the exterior angle theorem,

$$\angle ADB = \angle DBC + \angle DCB = 70^\circ + 70^\circ = 140^\circ.$$

Answer: (D) 140. □

Intermediate.

5. (2002 AMC 12A, Problem #4) Find the degree measure of an angle whose complement is 25% of its supplement.

- (A) 48 (B) 60 (C) 75 (D) 120 (E) 150

Strategy. Let the angle measure x° , and write its complement and supplement in terms of x .

Solution. The complement measures $(90 - x)^\circ$ and the supplement measures $(180 - x)^\circ$. Since 25% is $\frac{1}{4}$,

$$90 - x = \frac{180 - x}{4}.$$

Multiplying by 4 gives $360 - 4x = 180 - x$, so $3x = 180$ and $x = 60$. As a check, the complement is 30° , the supplement is 120° , and 30 is 25% of 120.

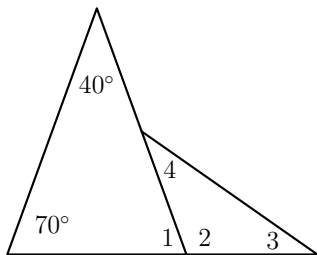
Answer: (B) 60. □

6. (1997 AJHSME, Problem #12)

$$\angle 1 + \angle 2 = 180^\circ$$

$$\angle 3 = \angle 4$$

Find $\angle 4$



- (A) 20° (B) 25° (C) 30° (D) 35° (E) 40°

Strategy. Start in the large triangle, where two angles are known, to find $\angle 1$. Then move to $\angle 2$ and the small triangle.

Solution. The large triangle has angles 70° , 40° and $\angle 1$, so

$$\angle 1 = 180^\circ - 70^\circ - 40^\circ = 70^\circ.$$

Since $\angle 1 + \angle 2 = 180^\circ$, $\angle 2 = 110^\circ$. The small triangle has angles $\angle 2$, $\angle 3$ and $\angle 4$, so

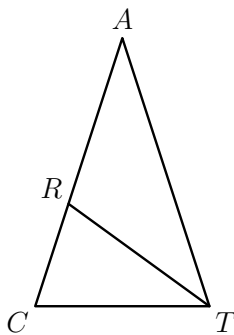
$$\angle 3 + \angle 4 = 180^\circ - 110^\circ = 70^\circ.$$

Since $\angle 3 = \angle 4$, each of them is 35° .

The exterior angle theorem gives the same result in one step: $\angle 1$ is an exterior angle of the small triangle, so $\angle 3 + \angle 4 = \angle 1 = 70^\circ$.

Answer: (D) 35° . □

7. (2000 AMC 8, Problem #13) In triangle CAT , we have $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$. If $\overline{TR}$ bisects $\angle ATC$, then $\angle CRT =$



- (A) 16° (B) 51° (C) 72° (D) 90° (E) 108°

Strategy. Find the two equal angles of triangle CAT from the angle sum. Then work in triangle CRT .

Solution. In triangle CAT ,

$$\angle ACT + \angle ATC = 180^\circ - 36^\circ = 144^\circ,$$

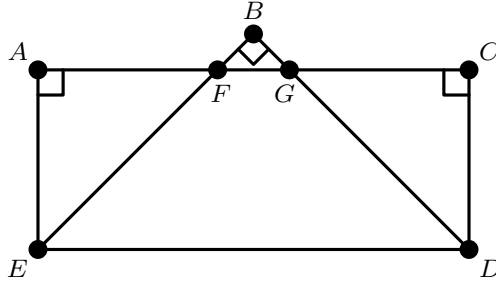
and the two angles are equal, so each measures 72° . Since $\overline{TR}$ bisects $\angle ATC$, $\angle RTC = \angle ATR = 36^\circ$. Point R lies on side $\overline{CA}$, so $\angle RCT = \angle ACT = 72^\circ$. In triangle CRT ,

$$\angle CRT = 180^\circ - 72^\circ - 36^\circ = 72^\circ.$$

Alternatively, $\angle CRT$ is the exterior angle of triangle ART at R , so $\angle CRT = \angle RAT + \angle ATR = 36^\circ + 36^\circ = 72^\circ$.

Answer: (C) 72° . □

8. (1995 AJHSME, Problem #13) In the figure, $\angle A$, $\angle B$ and $\angle C$ are right angles. If $\angle AEB = 40^\circ$ and $\angle BED = \angle BDE$, then $\angle CDE =$



- (A) 75° (B) 80° (C) 85° (D) 90° (E) 95°

Strategy. Find the two equal angles of triangle EBD first. Then use the angle sum of quadrilateral $ACDE$.

Solution. The right angle at B is the angle between lines EB and DB , so triangle EBD has $\angle EBD = 90^\circ$. Its other two angles add to 90° and are equal, so

$$\angle BED = \angle BDE = 45^\circ.$$

Ray EB lies inside $\angle AED$, so

$$\angle AED = \angle AEB + \angle BED = 40^\circ + 45^\circ = 85^\circ.$$

In quadrilateral $ACDE$, $\angle A = 90^\circ$, $\angle C = 90^\circ$ and $\angle E = 85^\circ$. The angles of a quadrilateral add to 360° , so

$$\angle CDE = 360^\circ - 90^\circ - 90^\circ - 85^\circ = 95^\circ.$$

Answer: (E) 95° . □

Advanced.

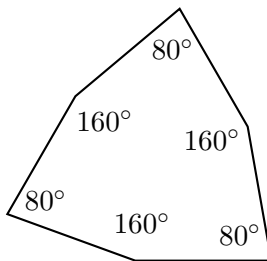
9. (1999 AHSME, Problem #7) What is the largest number of acute angles that a convex hexagon can have?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Strategy. Use the angle sum 720° and the fact that each angle of a convex polygon is less than 180° .

Solution. The interior angles of a hexagon add to $(6 - 2) \cdot 180^\circ = 720^\circ$. Suppose a convex hexagon had at least four acute angles. Four of the acute angles add to less than $4 \cdot 90^\circ = 360^\circ$, and the other two angles, each less than 180° , add to less than 360° . Then all six angles add to less than 720° , which is impossible. So a convex hexagon has at most three acute angles.

Three acute angles are possible. The convex hexagon below has angles 80° , 160° , 80° , 160° , 80° and 160° , which add to 720° .



Answer: (B) 3. □

10. (2007 AMC 10B, Problem #15) The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?
 (A) 125 (B) 144 (C) 153 (D) 173 (E) 180

Strategy. Write every angle in terms of $\angle A$ and use the angle sum of a quadrilateral.

Solution. Let $\angle A = a^\circ$. Then $\angle B = \frac{a}{2}$, $\angle C = \frac{a}{3}$ and $\angle D = \frac{a}{4}$ degrees. The angles of a quadrilateral add to 360° , so

$$a \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right) = 360.$$

Since $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{12+6+4+3}{12} = \frac{25}{12}$,

$$a = 360 \cdot \frac{12}{25} = 172.8.$$

Rounded to the nearest whole number, $\angle A$ measures 173° .

Answer: (D) 173. □

11. (1990 AHSME, Problem #3) The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is 75° , then the largest angle is
 (A) 95° (B) 100° (C) 105° (D) 110° (E) 115°

Strategy. Write the four angles in order around the trapezoid, starting from the smallest, and use the angle sum of a quadrilateral.

Solution. A trapezoid is a quadrilateral with a pair of parallel sides. In an arithmetic sequence, consecutive terms differ by the same amount d . The smallest angle is at one end of the sequence. Going around the trapezoid from the smallest angle, in the order of the sequence, the angles measure

$$75^\circ, \quad (75 + d)^\circ, \quad (75 + 2d)^\circ, \quad (75 + 3d)^\circ,$$

and the largest is $(75 + 3d)^\circ$. The angles of a quadrilateral add to 360° , so

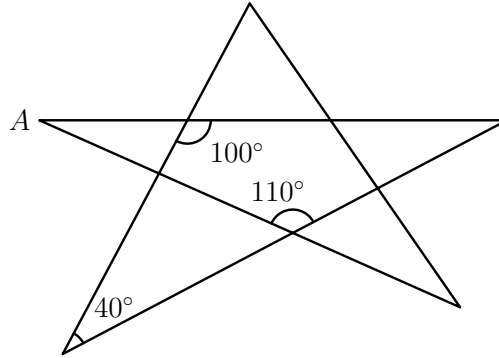
$$4 \cdot 75 + 6d = 360,$$

which gives $d = 10$. The largest angle measures $75^\circ + 30^\circ = 105^\circ$.

As a check, the angles in order are 75° , 85° , 95° and 105° . In a trapezoid, the two angles at the ends of each side that is not one of the parallel sides are same-side interior angles, so they add to 180° . Here $85 + 95 = 180$ and $105 + 75 = 180$.

Answer: (C) 105° . □

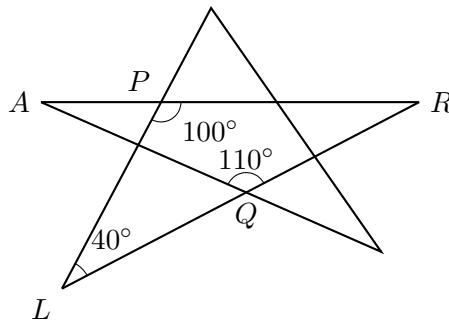
12. (1999 AMC 8, Problem #21) The degree measure of angle A is



- (A) 20 (B) 30 (C) 35 (D) 40 (E) 45

Strategy. Find a triangle that contains angle A and whose other two angles can be found from the given angles.

Solution. Label the points as in the figure below: P is the vertex of the 100° angle, Q is the vertex of the 110° angle, L is the vertex of the 40° angle, and R is the other end of the horizontal line through A .



In triangle LPR , $\angle PLR = 40^\circ$ and $\angle LPR = 100^\circ$, so

$$\angle PRL = 180^\circ - 40^\circ - 100^\circ = 40^\circ.$$

In triangle ARQ , the angle at R is this same angle, because P lies on $\overline{RA}$ and Q lies on $\overline{RL}$. So $\angle ARQ = 40^\circ$. The angle at Q is $\angle AQR = 110^\circ$. Therefore

$$\angle A = 180^\circ - 110^\circ - 40^\circ = 30^\circ.$$

Answer: (B) 30.

□

2. ANSWER KEY

Problem	Letter	Value
1	C	60
2	C	50°
3	D	7
4	D	140
5	B	60
6	D	35°
7	C	72°
8	E	95°
9	B	3
10	D	173
11	C	105°
12	B	30