SYSTEMATIC LISTING AND THE MULTIPLICATION PRINCIPLE FOR AMC 8: PRACTICE SET

ANDY JO

Contents

1. Problems 2
2. Answer Key 5

1. Problems

Beginner.

1. (1997 AJHSME, Problem #5) There are many two-digit multiples of 7, but only two of the multiples have a digit sum of 10. The sum of these two multiples of 7 is

(A) 119 **(B)** 126 **(C)** 140 **(D)** 175 **(E)** 189

2. (2004 AMC 10B, Problem #1) Each row of the Misty Moon Amphitheater has 33 seats. Rows 12 through 22 are reserved for a youth club. How many seats are reserved for this club?

(A) 297 **(B)** 330 **(C)** 363 **(D)** 396 **(E)** 726

3. (1988 AJHSME, Problem #19) What is the 100th number in the arithmetic sequence 1, 5, 9, 13, 17, 21, 25, ...?

(A) 397 **(B)** 399 **(C)** 401 **(D)** 403 **(E)** 405

4. (2021 Fall AMC 10A, Problem #6) Elmer the emu takes 44 equal strides to walk between consecutive telephone poles on a rural road. Oscar the ostrich can cover the same distance in 12 equal leaps. The telephone poles are evenly spaced, and the 41st pole along this road is exactly one mile (5280 feet) from the first pole. How much longer, in feet, is Oscar's leap than Elmer's stride?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 15

5. (1991 AJHSME, Problem #11) There are several sets of three different numbers whose sum is 15 which can be chosen from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many of these sets contain a 5?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Intermediate.

6. (2015 AMC 8, Problem #10) How many integers between 1000 and 9999 have four distinct digits?

(A) 3024 **(B)** 4536 **(C)** 5040 **(D)** 6480 **(E)** 6561

7. (2022 AMC 8, Problem #13) How many positive integers can fill the blank in the sentence below?
"One positive integer is _____ more than twice another, and the sum of the two numbers is 28"

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

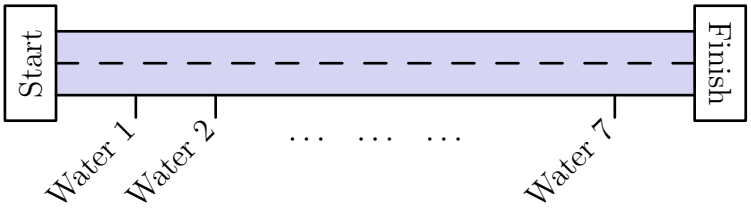
8. (2006 AMC 10A, Problem #9) How many sets of two or more consecutive positive integers have a sum of 15?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

9. (1994 AJHSME, Problem #24) A 2 by 2 square is divided into four 1 by 1 squares. Each of the small squares is to be painted either green or red. In how many different ways can the painting be accomplished so that no green square shares its top or right side with any red square? There may be as few as zero or as many as four small green squares.

(A) 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 16

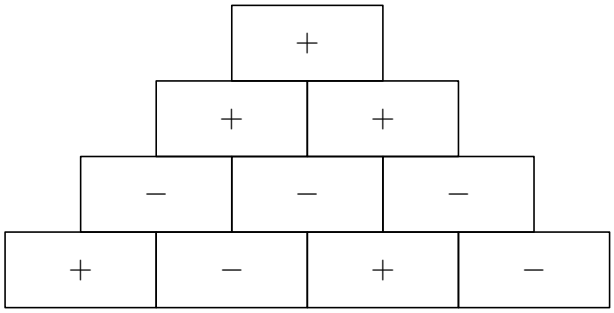
10. (2023 AMC 8, Problem #13) Along the route of a bicycle race, 7 water stations are evenly spaced between the start and finish lines, as shown in the figure below. There are also 2 repair stations evenly spaced between the start and finish lines. The 3rd water station is located 2 miles after the 1st repair station. How long is the race in miles?



(A) 8 **(B)** 16 **(C)** 24 **(D)** 48 **(E)** 96

Advanced.

11. (2018 AMC 8, Problem #19) In a sign pyramid a cell gets a "+" if the two cells below it have the same sign, and it gets a "-" if the two cells below it have different signs. The diagram below illustrates a sign pyramid with four levels. How many possible ways are there to fill the four cells in the bottom row to produce a "+" at the top of the pyramid?



(A) 2 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 16

12. (2022 AMC 10B, Problem #8) Consider the following 100 sets of 10 elements each:

$$\{1, 2, 3, \ldots, 10\},$$
$$\{11, 12, 13, \ldots, 20\},$$
$$\{21, 22, 23, \ldots, 30\},$$
$$\vdots$$
$$\{991, 992, 993, \ldots, 1000\}.$$

How many of these sets contain exactly two multiples of 7?

(A) 40 **(B)** 42 **(C)** 43 **(D)** 49 **(E)** 50

13. (2020 AMC 8, Problem #20) A scientist walking through a forest recorded as integers the heights of 5 trees standing in a row. She observed that each tree was either twice as tall or half as tall as the one to its right. Unfortunately some of her data was lost when rain fell on her notebook. Her notes are shown below, with blanks indicating the missing numbers. Based on her observations, the scientist was able to reconstruct the lost data. What was the average height of the trees, in meters?

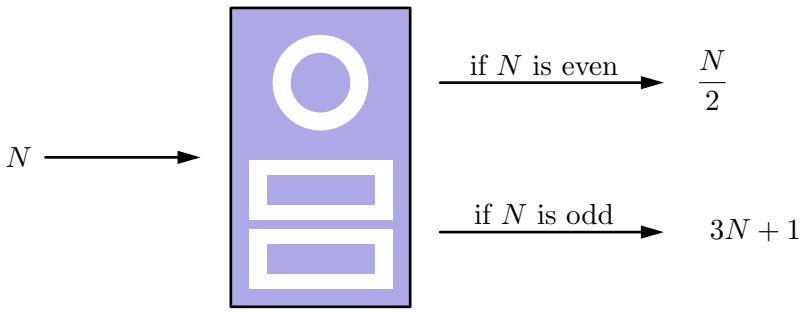
T_1	__ m
T_2	11 m
T_3	__ m
T_4	__ m
T_5	__ m
$\overline{T}$	__.2 m

(A) 22.2 **(B)** 24.2 **(C)** 33.2 **(D)** 35.2 **(E)** 37.2

14. (1990 AJHSME, Problem #25) How many different patterns can be made by shading exactly two of the nine unit squares in a 3×3 grid? Patterns that can be matched by flips and/or turns are not considered different.

(A) 3 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 18

15. (2020 AMC 8, Problem #22) When a positive integer N is fed into a machine, the output is a number calculated according to the rule shown below.



For example, starting with an input of $N = 7$, the machine will output $3 \cdot 7 + 1 = 22$. Then if the output is repeatedly inserted into the machine five more times, the final output is 26.

$$7 \to 22 \to 11 \to 34$$
$$\to 17 \to 52 \to 26$$

When the same 6-step process is applied to a different starting value of N, the final output is 1. What is the sum of all such integers N?

$$N \to __ \to __ \to __$$
$$\to __ \to __ \to 1$$

(A) 73 **(B)** 74 **(C)** 75 **(D)** 82 **(E)** 83

2. Answer Key

Problem	Letter	Value
1	A	119
2	C	363
3	A	397
4	B	8
5	B	4
6	B	4536
7	D	9
8	C	3
9	B	6
10	D	48
11	C	8
12	B	42
13	B	24.2
14	C	8
15	E	83