INTRODUCTION TO NUMBER THEORY FOR AMC 8: PRACTICE SET

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1. Problems

Beginner.

1. (1987 AJHSME, Problem #9) When finding the sum

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7},$$

what is the least common denominator used?

(A) 120 **(B)** 210 **(C)** 420 **(D)** 840 **(E)** 5040

2. (1990 AJHSME, Problem #4) Which of the following could **not** be the units digit [one's digit] of the square of a whole number?

(A) 1 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

3. (1994 AJHSME, Problem #10) For how many positive integer values of N ($N > 0$) is the expression

$$\frac{36}{N+2}$$

an integer?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

4. (2019 AMC 10B, Problem #2) Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

(A) 11 **(B)** 15 **(C)** 19 **(D)** 21 **(E)** 27

5. (2014 AMC 8, Problem #8) Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker $\$\underline{1A2}$. What is the missing digit A of this 3-digit number?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Intermediate.

6. (2020 AMC 10B, Problem #7) How many positive even multiples of 3 less than 2020 are perfect squares?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

7. (2013 AMC 8, Problem #13) When Clara totaled her scores, she inadvertently reversed the units digit and the tens digit of one score. By which of the following might her incorrect sum have differed from the correct one?

(A) 45 **(B)** 46 **(C)** 47 **(D)** 48 **(E)** 49

8. (2000 AMC 8, Problem #11) The number 64 has the property that it is divisible by its unit digit. How many whole numbers between 10 and 50 have this property?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 20

9. (2022 AMC 8, Problem #17) If n is an even positive integer, the double factorial notation $n!!$ represents the product of all the even integers from 2 to n. For example:

$$8!! = 2 \times 4 \times 6 \times 8$$

What is the units digit of the following sum?

$$2!! + 4!! + 6!! + \cdots + 2018!! + 2020!! + 2022!!$$

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

10. (2012 AMC 8, Problem #13) Jamar bought some pencils costing more than a penny each at the school bookstore and paid \$1.43. Sharona bought some of the same pencils and paid \$1.87. How many more pencils did Sharona buy than Jamar?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Advanced.

11. (2001 AMC 10, Problem #12) Suppose that n is the product of three consecutive integers and that n is divisible by 7. Which of the following is not necessarily a divisor of n?

(A) 6 **(B)** 14 **(C)** 21 **(D)** 28 **(E)** 42

12. (2006 AMC 8, Problem #23) A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5

13. (2019 AMC 8, Problem #23) After Euclid High School's last basketball game, it was determined that $\frac{1}{4}$ of the team's points were scored by Alexa and $\frac{2}{7}$ were scored by Brittany. Chelsea scored 15 points. None of the other 7 team members scored more than 2 points. What was the total number of points scored by the other 7 team members?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

14. (2015 AMC 8, Problem #22) On June 1, a group of students is standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

(A) 21 **(B)** 30 **(C)** 60 **(D)** 90 **(E)** 1080

15. (2011 AMC 8, Problem #22) What is the tens digit of 7^{2011}?

(A) 0 **(B)** 1 **(C)** 3 **(D)** 4 **(E)** 7

2. Answer Key

Problem	Letter	Value
1	C	420
2	E	8
3	A	7
4	E	27
5	D	3
6	A	7
7	A	45
8	C	17
9	B	2
10	C	4
11	D	28
12	A	0
13	B	11
14	C	60
15	D	4