PERCENTS AND PERCENT CHANGE FOR AMC 8: SOLUTIONS

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1. Solutions

Beginner.

1. (1985 AJHSME, Problem #14) The difference between a 6.5% sales tax and a 6% sales tax on an item priced at \$20 before tax is

(A) \$0.01 **(B)** \$0.10 **(C)** \$0.50 **(D)** \$1 **(E)** \$10

Strategy. Find each tax as a percent of the \$20 price, then subtract.

Solution. The 6.5% tax on \$20 is $0.065 \cdot 20 = \$1.30$, and the 6% tax is $0.06 \cdot 20 = \$1.20$, so the difference between the taxes is $1.30 - 1.20 = 0.10$ dollars. The rule for two percents of the same quantity gives this in one step: the difference is $(6.5 - 6)\% = 0.5\%$ of \$20, which is half of 1% of \$20, or half of \$0.20.

Answer: (B) \$0.10. □

2. (2003 AMC 8, Problem #5) If 20% of a number is 12, what is 30% of the same number?

(A) 15 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

Strategy. Find the number from the part that is known, then take 30% of the number.

Solution. Since 12 is 20% of the number, 1% of the number is $\frac{12}{20} = 0.6$, and the number is $100 \cdot 0.6 = 60$. Then 30% of 60 is $0.3 \cdot 60 = 18$.

The number itself is not needed: 30% is $\frac{30}{20} = 1.5$ times 20%, so 30% of the number is $1.5 \cdot 12 = 18$.

Answer: (B) 18. □

3. (2001 AMC 8, Problem #10) A collector offers to buy state quarters for 2000% of their face value. At that rate how much will Bryden get for his four state quarters?

(A) \$20 **(B)** \$50 **(C)** \$200 **(D)** \$500 **(E)** \$2000

Strategy. A percent greater than 100 gives more than the whole: 2000% of an amount is $\frac{2000}{100} = 20$ times the amount.

Solution. Four quarters have a face value of $4 \cdot \$0.25 = \$1$. The collector pays 2000% of this face value, which is

$$\frac{2000}{100} \cdot 1 = 20$$

dollars.

Answer: (A) \$20. □

4. (2002 AMC 8, Problem #14) A merchant offers a large group of items at 30% off. Later, the merchant takes 20% off these sale prices and claims that the final price of these items is 50% off the original price. The total discount is

(A) 35% **(B)** 44% **(C)** 50% **(D)** 56% **(E)** 60%

Strategy. Successive discounts multiply their multipliers. Measure the total discount against the original price.

Solution. The first discount multiplies each price by $1 - 0.30 = 0.7$, and the second multiplies each sale price by $1 - 0.20 = 0.8$. So the final price is

$$0.7 \cdot 0.8 = 0.56$$

times the original price, which is 56% of it, and the total discount is $100\% - 56\% = 44\%$. The merchant's 50% simply adds 30% and 20%, but the second discount removes 20% of

70% of each original price, which is only $0.2 \cdot 0.7 = 0.14 = 14\%$ of it. The two discounts together remove $30\% + 14\% = 44\%$.

Answer: (B) 44%. □

Intermediate.

5. (2009 AMC 10B, Problem #8) In a certain year the price of gasoline rose by 20% during January, fell by 20% during February, rose by 25% during March, and fell by x% during April. The price of gasoline at the end of April was the same as it had been at the beginning of January. To the nearest integer, what is x?

(A) 12 **(B)** 17 **(C)** 20 **(D)** 25 **(E)** 35

Strategy. By the corollary, January and February together lower the price by a known percent. March and April together reverse that change, and the March multiplier is known.

Solution. By the corollary with $p = 20$, the rise in January and the fall in February together lower the price by $20^2/100 = 4$ percent, which is a multiplier of 0.96. At the end of April the price is back at its starting value, so by the proposition on reversing a percent change, March and April together multiply it by $\frac{1}{0.96} = \frac{25}{24}$. March multiplies it by $1.25 = \frac{5}{4}$, so by the theorem on successive percent changes, April multiplies it by $\frac{25}{24} \div \frac{5}{4} = \frac{5}{6}$. That is a decrease of $\frac{1}{6}$, or $16\frac{2}{3}$%, so $x = 16\frac{2}{3}$, which rounds to 17.

Answer: (B) 17. □

6. (1998 AJHSME, Problem #14) At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

(A) 12 **(B)** 15 **(C)** 30 **(D)** 36 **(E)** 40

Strategy. Both percents describe one group, the students in both clubs. Find its size from the Science Club, then find the Math Club as the whole of which it is 30%.

Solution. The Science Club has 15 members, and 80% of them, $0.8 \cdot 15 = 12$ students, belong to both clubs. In the Math Club these students are the part $P = 12$ with $p = 30$, so by the proposition on finding the whole the Math Club has

$$N = \frac{100 \cdot 12}{30} = 40$$

students. As a check, 30% of 40 is 12.

Answer: (E) 40. □

7. (1969 AHSME, Problem #2) If an item is sold for x dollars, there is a loss of 15% based on the cost. If, however, the same item is sold for y dollars, there is a profit of 15% based on the cost. The ratio $y : x$ is:

(A) $23 : 17$ **(B)** $17y : 23$ **(C)** $23x : 17$ **(D)** dependent upon the cost
(E) none of these

Strategy. The ratio of the two selling prices is the ratio of their multipliers.

Solution. A loss of 15% based on the cost means that x is $1 - 0.15 = 0.85$ times the cost. A profit of 15% based on the cost is a markup of 15%, so y is 1.15 times the cost. The cost cancels in the ratio, and multiplying both terms by 20 gives

$$y : x = 1.15 : 0.85 = 23 : 17.$$

The ratio is the same for every cost.

Answer: (A) $23 : 17$. □

8. (1997 AJHSME, Problem #16) Penni Precisely buys \$100 worth of stock in each of three companies: Alabama Almonds, Boston Beans, and California Cauliflower. After one year, AA was up 20%, BB was down 25%, and CC was unchanged. For the second year, AA was down 20% from the previous year, BB was up 25% from the previous year, and CC was unchanged. If A, B, and C are the final values of the stock, then

(A) $A = B = C$ **(B)** $A = B < C$ **(C)** $C < B = A$ **(D)** $A < B < C$ **(E)** $B < A < C$

Strategy. Find each final value with multipliers.

Solution. Each change multiplies the value of a stock by its multiplier, so the final values are

$$A = 100 \cdot 1.2 \cdot 0.8 = 96, \qquad B = 100 \cdot 0.75 \cdot 1.25 = 93.75, \qquad C = 100.$$

Hence $B < A < C$. The corollary on an increase and a decrease by the same percent explains the order: over the two years AA lost 4% of its value and BB lost 6.25%, since

$$\frac{20^2}{100} = 4 \qquad \text{and} \qquad \frac{25^2}{100} = 6.25.$$

Answer: (E) $B < A < C$. □

Advanced.

9. (2019 AMC 8, Problem #22) A store increased the original price of a shirt by a certain percent and then decreased the new price by the same percent. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased?

(A) 16 **(B)** 20 **(C)** 28 **(D)** 36 **(E)** 40

Strategy. By the corollary on an increase and a decrease by the same percent, raising a price by p% and then lowering it by p% lowers it by $p^2/100$ percent.

Solution. Let the price be increased and then decreased by p%. The resulting price is 84% of the original price, so the two changes together are a decrease of 16%. By the corollary, this decrease is $p^2/100$ percent, so

$$\frac{p^2}{100} = 16, \qquad p^2 = 1600, \qquad p = 40.$$

As a check, $1.4 \cdot 0.6 = 0.84$.

Answer: (E) 40. □

10. (1971 AHSME, Problem #4) After simple interest for two months at 5% per annum was credited, a Boy Scout Troop had a total of \$255.31 in the Council Treasury. The interest credited was a number of dollars plus the following number of cents:

(A) 11 **(B)** 12 **(C)** 13 **(D)** 21 **(E)** 31

Strategy. Two months is $\frac{1}{6}$ of a year. Write the interest and the total as fractions of the principal; then the interest is a known fraction of the total.

Solution. Let the principal be P dollars. Two months is $\frac{2}{12} = \frac{1}{6}$ of a year, so the interest is

$$P \cdot \frac{5}{100} \cdot \frac{1}{6} = \frac{P}{120}.$$

The total is $P + \frac{P}{120} = \frac{121}{120}P$, so the interest $\frac{P}{120}$ is $\frac{1}{121}$ of the total, and the interest is

$$\frac{255.31}{121} = 2.11$$

dollars, because $121 \cdot 2 = 242$ and $121 \cdot 0.11 = 13.31$, which add to 255.31. The interest was 2 dollars plus 11 cents. As a check, the principal was $255.31 - 2.11 = \$253.20$, and $253.20/120 = 2.11$.

Answer: (A) 11. □

11. (1956 AHSME, Problem #4) A man has \$10,000 to invest. He invests \$4,000 at 5% and \$3,500 at 4%. In order to have a yearly income of \$500, he must invest the remainder at:
(A) 6% **(B)** 6.1% **(C)** 6.2% **(D)** 6.3% **(E)** 6.4%

Strategy. At 5%, the whole \$10,000 would earn exactly the \$500 wanted. Find how much the \$3,500 at 4% falls short, and make the remainder earn that much more than 5%.

Solution. If every dollar earned 5%, the \$10,000 would bring in $0.05 \cdot 10{,}000 = \$500$ a year, the income wanted. The \$4,000 does earn 5%. The \$3,500 earns 4% instead, and by the rule for two percents of the same quantity, 4% of \$3,500 falls short of 5% of it by 1% of \$3,500, which is \$35. So the other $10{,}000 - 4{,}000 - 3{,}500 = \$2{,}500$ must earn \$35 more than 5% of itself. Since \$35 is

$$\frac{35}{2{,}500} \cdot 100\% = 1.4\%$$

of \$2,500, the remainder must be invested at $5\% + 1.4\% = 6.4\%$. As a check, the three amounts then earn $200 + 140 + 160 = 500$ dollars.

Answer: (E) 6.4%. □

12. (2005 AMC 8, Problem #22) A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three sizes from best to worst buy.
(A) SML **(B)** LMS **(C)** MSL **(D)** LSM **(E)** MLS

Strategy. Compare every box with the small box: write its cost and its amount of detergent as multiples of the small box's. The better buy has the lower cost per unit of detergent.

Solution. In amount of detergent, the large box holds 2 times as much as the small box, and the medium box, with 20% less detergent than the large box, holds $0.8 \cdot 2 = 1.6$ times as much. The medium box costs 1.5 times as much as the small box, and the large box, priced 30% above the medium box, costs $1.3 \cdot 1.5 = 1.95$ times as much. Dividing cost by amount gives each box's cost per unit of detergent as a multiple of the small box's:

$$M\colon \frac{1.5}{1.6} = 0.9375, \qquad L\colon \frac{1.95}{2} = 0.975.$$

Both are below the small box's 1, and $0.9375 < 0.975$, so the ranking from best to worst is M, L, S.

Answer: (E) MLS. □

2. Answer Key

Problem	Letter	Value
1	B	\$0.10
2	B	18
3	A	\$20
4	B	44%
5	B	17
6	E	40
7	A	$23 : 17$
8	E	$B < A < C$
9	E	40
10	A	11
11	E	6.4%
12	E	MLS