PERCENTS AND PERCENT CHANGE FOR AMC 8: PRACTICE SET SOLUTIONS

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1. Solutions

Beginner.

1. (2020 AMC 8, Problem #5) Three fourths of a pitcher is filled with pineapple juice. The pitcher is emptied by pouring an equal amount of juice into each of 5 cups. What percent of the total capacity of the pitcher did each cup receive?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

Strategy. Find the fraction of the whole pitcher that goes into one cup, then rewrite that fraction with denominator 100.

Solution. The juice fills $\frac{3}{4}$ of the pitcher and is shared equally among 5 cups, so each cup receives

$$\frac{3}{4} \div 5 = \frac{3}{20}$$

of the pitcher. Rewritten with denominator 100, this fraction is $\frac{3}{20} = \frac{15}{100} = 15\%$. As a check, the five cups together receive $5 \cdot 15\% = 75\%$ of the pitcher, which is $\frac{3}{4}$ of it.

Answer: (C) 15. □

2. (1989 AJHSME, Problem #21) Jack had a bag of 128 apples. He sold 25% of them to Jill. Next he sold 25% of those remaining to June. Of those apples still in his bag, he gave the shiniest one to his teacher. How many apples did Jack have then?

(A) 7 **(B)** 63 **(C)** 65 **(D)** 71 **(E)** 111

Strategy. Each sale takes 25% of the apples in the bag at that moment, so the two sales are successive percent changes, each with multiplier 0.75. The apple for the teacher is subtracted at the end.

Solution. Selling 25% of the apples in the bag leaves 75% of them, so each sale multiplies the number of apples by $1 - 0.25 = 0.75$. By the theorem on successive percent changes, the two sales together multiply it by $0.75 \cdot 0.75 = 0.5625$, which leaves

$$128 \cdot 0.5625 = 72$$

apples. Step by step, Jill buys 32 apples and 96 remain, and then June buys 24 and 72 remain. After Jack gives one apple to his teacher, he has $72 - 1 = 71$ apples. The two sales remove 56 apples, which is 43.75% of the bag rather than 50%, because the second 25% is taken of only 96 apples.

Answer: (D) 71. □

3. (2007 AMC 8, Problem #6) The average cost of a long-distance call in the USA in 1985 was 41 cents per minute, and the average cost of a long-distance call in the USA in 2005 was 7 cents per minute. Find the approximate percent decrease in the cost per minute of a long-distance call.

(A) 7 **(B)** 17 **(C)** 34 **(D)** 41 **(E)** 80

Strategy. A percent decrease is the drop divided by the original value, and here the original value is the 1985 cost of 41 cents.

Solution. The cost per minute fell from 41 cents to 7 cents, a drop of $41 - 7 = 34$ cents. By the definition of a percent decrease, with the 1985 cost as the original value, the decrease is

$$\frac{34}{41} \cdot 100\% \approx 82.9\%.$$

Since $0.8 \cdot 41 = 32.8$ and $0.85 \cdot 41 = 34.85$, the drop of 34 cents is between 80% and 85% of the original cost, and the closest choice is 80. The drop of 34 cents is not a decrease of 34%, because a percent decrease divides the drop by the original cost.

Answer: (E) 80. □

4. (2010 AMC 8, Problem #9) Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

(A) 63 **(B)** 75 **(C)** 80 **(D)** 84 **(E)** 86

Strategy. Each percent is a percent of its own test. Count the correct answers on each test, then write the total as a percent of all the problems.

Solution. By the proposition on a percent of a quantity, Ryan answered $0.8 \cdot 25 = 20$ problems correctly on the first test, $0.9 \cdot 40 = 36$ on the second, and $0.7 \cdot 10 = 7$ on the third. In all he answered $20 + 36 + 7 = 63$ of the $25 + 40 + 10 = 75$ problems correctly, and

$$\frac{63}{75} = \frac{21}{25} = \frac{84}{100} = 84\%.$$

The answer is not $\frac{80+90+70}{3} = 80$, because the tests have different numbers of problems, and the 40-problem test, with the highest percent, has the most problems.

Answer: (D) 84. □

5. (2023 AMC 10B, Problem #2) Carlos went to a sports store to buy running shoes. Running shoes were on sale, with prices reduced by 20% on every pair of shoes. Carlos also knew that he had to pay a 7.5% sales tax on the discounted price. He had 43 dollars. What is the original (before discount) price of the most expensive shoes he could afford to buy?

(A) \$46 **(B)** \$50 **(C)** \$48 **(D)** \$47 **(E)** \$49

Strategy. Taking 20% off leaves 80% of the original price, and a 7.5% tax on that amount brings the total to 107.5% of it. The total paid must be at most 43 dollars.

Solution. If the original price is p dollars, Carlos pays $1.075 \cdot 0.8 \cdot p = 0.86p$ dollars. He can afford the shoes when $0.86p \leq 43$, that is, $p \leq \frac{43}{0.86} = 50$. The most expensive shoes he can afford had an original price of \$50.

Answer: (B) \$50. □

Intermediate.

6. (2000 AMC 8, Problem #10) Ara and Shea were once the same height. Since then Shea has grown 20% while Ara has grown half as many inches as Shea. Shea is now 60 inches tall. How tall, in inches, is Ara now?

(A) 48 **(B)** 51 **(C)** 52 **(D)** 54 **(E)** 55

Strategy. Shea's growth of 20% is a change with multiplier 1.2 that ends at 60 inches. Divide by the multiplier to find the height that Ara and Shea once shared.

Solution. Growing 20% multiplies Shea's height by 1.2, so by the proposition on reversing a percent change, Shea was once

$$\frac{60}{1.2} = 50$$

inches tall. Shea has grown $60-50=10$ inches, and Ara has grown half as many, 5 inches. Ara also started at 50 inches, so Ara is now $50+5=55$ inches tall. Taking 20% of 60 inches instead would measure Shea's growth against the wrong base, the new height.

Answer: (E) 55. □

7. (1951 AHSME, Problem #5) Mr. A owns a home worth \$10,000. He sells it to Mr. B at a 10% profit based on the worth of the house. Mr. B sells the house back to Mr. A at a 10% loss. Then:

(A) A comes out even **(B)** A makes \$1100 on the deal
(C) A makes \$1000 on the deal **(D)** A loses \$900 on the deal
(E) A loses \$1000 on the deal

Strategy. The two percents have different bases: A's profit is 10% of the \$10,000 worth, and B's loss is 10% of the price B paid. Compare the money A receives with the money A pays.

Solution. A profit of 10% based on the worth of the house is a markup of 10%, so B pays $1.1\cdot 10{,}000 = \$11{,}000$. The problem does not say what B's loss is based on, so it is based on B's cost, \$11,000, and a loss of 10% gives a selling price of $0.9\cdot 11{,}000 = \$9900$. In the end A owns the house again, having received \$11,000 and paid \$9900, so A makes $11{,}000-9900=\$1100$. The two changes of 10% do not cancel, because the loss is 10% of the larger amount.

Answer: (B) A makes \$1100 on the deal. □

8. (2010 AMC 8, Problem #15) A jar contains 5 different colors of gum drops. 30% are blue, 20% are brown, 15% are red, 10% are yellow, and the other 30 gum drops are green. If half of the blue gum drops are replaced by brown gum drops, how many gum drops will be brown?

(A) 35 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 64

Strategy. The four colors given as percents make up a known part of the jar, so the 30 green drops are the rest. Find the whole jar from this part, then count the blue and brown drops.

Solution. Blue, brown, red, and yellow drops make up $30\%+20\%+15\%+10\%=75\%$ of the jar, so the 30 green drops are the other 25%. By the proposition on finding the whole, with $P=30$ and $p=25$, the jar holds

$$N=\frac{100\cdot 30}{25}=120$$

gum drops. Of these, $0.3\cdot 120=36$ are blue and $0.2\cdot 120=24$ are brown. Half of the blue drops is 18 drops, and replacing them by brown drops gives $24+18=42$ brown drops.

Answer: (C) 42. □

9. (2008 AMC 10A, Problem #12) In a collection of red, blue, and green marbles, there are 25% more red marbles than blue marbles, and there are 60% more green marbles than red marbles. Suppose that there are r red marbles. What is the total number of marbles in the collection?

(A) $2.85r$ **(B)** $3r$ **(C)** $3.4r$ **(D)** $3.85r$ **(E)** $4.25r$

Strategy. Write each comparison with a multiplier. The red count is 1.25 times the blue count, so the blue count is the red count divided by 1.25, and the green count is 1.6 times the red count.

Solution. Let b be the number of blue marbles. Written with a multiplier, 25% more red marbles than blue marbles means $r = 1.25b$, so by the proposition on reversing a percent change,

$$b = \frac{r}{1.25} = \frac{4}{5}r = 0.8r.$$

In the same way, 60% more green marbles than red marbles means that there are $1.6r$ green marbles. The total number of marbles is

$$r + 0.8r + 1.6r = 3.4r.$$

The blue count $0.8r$ is 20% less than r, not 25% less, because the 25% is a percent of the blue count.

Answer: (C) $3.4r$. □

10. (2013 AMC 10A, Problem #10) A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

(A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

Strategy. Split the flowers by color. The carnations of each color are a fraction of that color, which is itself a percent of the bouquet, so multiply the two.

Solution. Six tenths of the flowers, that is 60%, are pink, so the other 40% are red. One third of the pink flowers are roses, so two thirds of them are carnations. A fraction of a part that is a percent of the whole is found by multiplying, so the pink carnations make up

$$\frac{2}{3} \cdot 60\% = 40\%$$

of the bouquet. Three fourths of the red flowers are carnations, so the red carnations make up $\frac{3}{4} \cdot 40\% = 30\%$ of the bouquet. All the carnations together make up $40\% + 30\% = 70\%$ of the flowers.

Answer: (E) 70. □

Advanced.

11. (1952 AHSME, Problem #16) If the base of a rectangle is increased by 10% and the area is unchanged, then the altitude is decreased by:

(A) 9% **(B)** 10% **(C)** 11% **(D)** $11\frac{1}{9}\%$ **(E)** $9\frac{1}{11}\%$

Strategy. The area is the base times the altitude. The base is multiplied by 1.1, so the altitude must be multiplied by the multiplier that undoes this change.

Solution. The area of a rectangle is its base times its altitude. Increasing the base by 10% multiplies it by $1.1 = \frac{11}{10}$. If the altitude is multiplied by m, the area is multiplied by $\frac{11}{10}m$, so the area is unchanged exactly when $\frac{11}{10}m = 1$, that is, when

$$m = \frac{1}{1.1} = \frac{10}{11}.$$

This is the multiplier that undoes an increase of 10%, as in the proposition on reversing a percent change. Since $\frac{10}{11} = 1 - \frac{1}{11}$, the altitude is decreased by

$$\frac{1}{11} \cdot 100\% = \frac{100}{11}\% = 9\frac{1}{11}\%.$$

For example, a rectangle with base 10 and altitude 11 and a rectangle with base 11 and altitude 10 both have area 110, and the altitude falls by 1 out of 11. A decrease of 10% would not keep the area: by the corollary on an increase and a decrease by the same percent, the area would be multiplied by $1 - 0.1^2 = 0.99$.

Answer: (E) $9\frac{1}{11}\%$. □

12. (1974 AHSME, Problem #7) A town's population increased by 1,200 people, and then this new population decreased by 11%. The town now had 32 less people than it did before the 1,200 increase. What is the original population?

(A) 1,200 **(B)** 11,200 **(C)** 9,968 **(D)** 10,000 **(E)** none of these

Strategy. Compare the population before and after the decrease with the original population. The decrease removed $1{,}200 + 32$ people, and these are 11% of the population after the increase.

Solution. After the increase, the town had 1,200 more people than at first, and after the decrease it had 32 fewer people than at first. So the decrease removed $1{,}200 + 32 = 1{,}232$ people. The decrease of 11% is a percent of the new population, the one after the increase, so 1,232 is 11% of that population. By the proposition on finding the whole, with $P = 1{,}232$ and $p = 11$, the population after the increase was

$$N = \frac{100 \cdot 1{,}232}{11} = 100 \cdot 112 = 11{,}200,$$

and the original population was $11{,}200 - 1{,}200 = 10{,}000$. As a check, $0.89 \cdot 11{,}200 = 9{,}968$, which is 32 less than 10,000.

Answer: (D) 10,000. □

13. (1953 AHSME, Problem #17) A man has part of \$4500 invested at 4% and the rest at 6%. If his annual return on each investment is the same, the average rate of interest which he realizes on the \$4500 is:

(A) 5% **(B)** 4.8% **(C)** 5.2% **(D)** 4.6% **(E)** none of these

Strategy. Equal yearly returns mean that 4% of one part equals 6% of the other, which fixes the ratio of the two parts. Find the parts, then write the total interest as a percent of \$4500.

Solution. Let a dollars be invested at 4% and b dollars at 6%, so $a + b = 4500$. One year of simple interest is $0.04a$ dollars on the first part and $0.06b$ dollars on the second. These are equal, so $4a = 6b$, and by cross-multiplication

$$\frac{a}{b} = \frac{6}{4} = \frac{3}{2}.$$

So the two parts are in the ratio $3 : 2$. The \$4500 is $3 + 2 = 5$ parts of $4500 \div 5 = 900$ dollars each, so $a = 3 \cdot 900 = 2700$ and $b = 2 \cdot 900 = 1800$. Each part earns $0.04 \cdot 2700 = 0.06 \cdot 1800 = 108$ dollars a year, so the whole \$4500 earns \$216 a year. The average rate on the \$4500 is the rate that gives this interest on the whole amount, which is

$$\frac{216}{4500} \cdot 100\% = 4.8\%.$$

It is less than 5%, the rate halfway between 4% and 6%, because more of the money earns the lower rate.

Answer: (B) 4.8%. □

14. (1955 AHSME, Problem #22) On a \$10,000 order a merchant has a choice between three successive discounts of 20%, 20%, and 10% and three successive discounts of 40%, 5%, and 5%. By choosing the better offer, he can save:

(A) nothing at all **(B)** \$440 **(C)** \$330 **(D)** \$345 **(E)** \$360

Strategy. Successive discounts multiply, so each offer multiplies the price by the product of its three multipliers. Both lists of discounts add up to 50%, but the products differ.

Solution. A discount of $p\%$ multiplies the price by $1-\frac{p}{100}$, and by the theorem on successive percent changes, three discounts in a row multiply it by the product of their multipliers. The first offer multiplies the price by

$$0.8 \cdot 0.8 \cdot 0.9 = 0.576,$$

so the merchant pays $0.576 \cdot 10{,}000 = \$5760$. The second offer multiplies it by

$$0.6 \cdot 0.95 \cdot 0.95 = 0.6 \cdot 0.9025 = 0.5415,$$

so the merchant pays \$5415. The second offer is the better one, and it saves the merchant $5760 - 5415 = 345$ dollars. Neither offer takes 50% off, since every discount after the first is a percent of a price that is already lower.

Answer: (D) \$345. □

15. (2007 AMC 8, Problem #20) Before district play, the Unicorns had won 45% of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the Unicorns play in all?

(A) 48 **(B)** 50 **(C)** 52 **(D)** 54 **(E)** 60

Strategy. Before district play, the losses exceeded the wins by $55\% - 45\% = 10\%$ of the games played. District play closed this gap exactly, so 10% of the earlier games is a known number of games.

Solution. Let n be the number of games played before district play. The Unicorns won 45% of them and lost the other 55%, so by the rule for two percents of the same quantity, their losses exceeded their wins by $(55 - 45)\% = 10\%$ of n. In district play they won 6 games and lost 2, which cut this gap by $6 - 2 = 4$ games. They finished with half their games won, so their wins and losses were then equal, and the gap of 10% of n was exactly 4 games. By the proposition on finding the whole, with $P = 4$ and $p = 10$,

$$n = \frac{100 \cdot 4}{10} = 40.$$

In all the Unicorns played $40 + 8 = 48$ games. As a check, they won $0.45 \cdot 40 = 18$ of their first 40 games and $18 + 6 = 24$ of all 48, which is half.

Answer: (A) 48. □

2. Answer Key

Problem	Letter	Value
1	C	15
2	D	71
3	E	80
4	D	84
5	B	\$50
6	E	55
7	B	A makes \$1100 on the deal
8	C	42
9	C	$3.4r$
10	E	70
11	E	$9\frac{1}{11}\%$
12	D	10,000
13	B	4.8%
14	D	\$345
15	A	48