

RATIOS, RATES, AND PROPORTIONS FOR AMC 8: SOLUTIONS

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1. SOLUTIONS

Beginner.

1. (2004 AMC 8, Problem #1) On a map, a 12-centimeter length represents 72 kilometers. How many kilometers does a 17-centimeter length represent?

(A) 6 (B) 102 (C) 204 (D) 864 (E) 1224

Strategy. Find how many kilometers one centimeter represents.

Solution. Since 12 centimeters represent 72 kilometers, 1 centimeter represents $\frac{72}{12} = 6$ kilometers. So 17 centimeters represent $17 \cdot 6 = 102$ kilometers.

Answer: (B) 102. □

2. (2014 AMC 8, Problem #5) Margie's car can go 32 miles on a gallon of gas, and gas currently costs \$4 per gallon. How many miles can Margie drive on \$20 worth of gas?

(A) 64 (B) 128 (C) 160 (D) 320 (E) 640

Strategy. Find how many gallons \$20 buys, then use the number of miles per gallon.

Solution. At \$4 per gallon, \$20 buys $\frac{20}{4} = 5$ gallons of gas. At 32 miles per gallon, 5 gallons last for $5 \cdot 32 = 160$ miles. Equivalently, the car goes $\frac{32}{4} = 8$ miles per dollar, and $8 \cdot 20 = 160$.

Answer: (C) 160. □

3. (2004 AMC 8, Problem #3) Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they shared, how many meals should they have ordered to have just enough food for the 12 of them?

(A) 8 (B) 9 (C) 10 (D) 15 (E) 18

Strategy. The ratio of meals to people fed stays the same. Set up a proportion.

Solution. Twelve meals feed 18 people, so the ratio of meals to people is $12 : 18 = 2 : 3$. If m meals feed 12 people, then

$$\frac{m}{12} = \frac{2}{3} \quad \Rightarrow \quad 3m = 24 \quad \Rightarrow \quad m = 8.$$

Equivalently, each meal feeds $\frac{18}{12} = 1\frac{1}{2}$ people, so 12 people need $\frac{12}{1\frac{1}{2}} = 8$ meals.

Answer: (A) 8. □

4. (2020 AMC 8, Problem #3) Carrie has a rectangular garden that measures 6 feet by 8 feet. She plants the entire garden with strawberry plants. Carrie is able to plant 4 strawberry plants per square foot, and she harvests an average of 10 strawberries per plant. How many strawberries can she expect to harvest?

(A) 560 (B) 960 (C) 1120 (D) 1920 (E) 3840

Strategy. Use the two unit rates one after the other: square feet to plants, then plants to strawberries.

Solution. The garden has an area of $6 \cdot 8 = 48$ square feet. At 4 plants per square foot, it holds $48 \cdot 4 = 192$ plants. At 10 strawberries per plant, Carrie can expect $192 \cdot 10 = 1920$ strawberries.

Answer: (D) 1920. □

Intermediate.

5. (2014 AMC 8, Problem #7) There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of the number of girls to the number of boys in her class?
 (A) 3 : 4 (B) 4 : 3 (C) 3 : 2 (D) 7 : 4 (E) 2 : 1

Strategy. Set aside the four extra girls. The rest of the class has equal numbers of boys and girls.

Solution. Without the 4 extra girls, the other $28 - 4 = 24$ students are half boys and half girls, 12 of each. So the class has 12 boys and $12 + 4 = 16$ girls, and the ratio of girls to boys is $16 : 12 = 4 : 3$.

Answer: (B) 4 : 3. □

6. (2013 AMC 8, Problem #7) Trey and his mom stopped at a railroad crossing to let a train pass. As the train began to pass, Trey counted 6 cars in the first 10 seconds. It took the train 2 minutes and 45 seconds to clear the crossing at a constant speed. Which of the following was the most likely number of cars in the train?
 (A) 60 (B) 80 (C) 100 (D) 120 (E) 140

Strategy. Find the rate in cars per second, then multiply by the time in seconds.

Solution. Six cars in 10 seconds is a rate of $\frac{6}{10} = 0.6$ car per second. The train took $2 \cdot 60 + 45 = 165$ seconds, so it had about $0.6 \cdot 165 = 99$ cars. The closest choice is 100.

Answer: (C) 100. □

7. (2022 AMC 8, Problem #7) When the World Wide Web first became popular in the 1990s, download speeds reached a maximum of about 56 kilobits per second. Approximately how many minutes would the download of a 4.2-megabyte song have taken at that speed? (Note that there are 8000 kilobits in a megabyte.)
 (A) 0.6 (B) 10 (C) 1800 (D) 7200 (E) 36000

Strategy. Convert the size of the song to kilobits, divide by the rate to get seconds, then convert the seconds to minutes.

Solution. The song is $4.2 \cdot 8000 = 33,600$ kilobits. At 56 kilobits per second, the download takes $\frac{33,600}{56} = 600$ seconds, which is $\frac{600}{60} = 10$ minutes.

Answer: (B) 10. □

8. (2016 AMC 8, Problem #12) Jefferson Middle School has the same number of boys and girls. Three-fourths of the girls and two-thirds of the boys went on a field trip. What fraction of the students on the field trip were girls?
 (A) $\frac{1}{2}$ (B) $\frac{9}{17}$ (C) $\frac{7}{13}$ (D) $\frac{2}{3}$ (E) $\frac{14}{15}$

Strategy. The answer does not depend on the size of the school, so pick a convenient number of boys and girls that both 3 and 4 divide.

Solution. Suppose the school has 12 boys and 12 girls. Then $\frac{3}{4} \cdot 12 = 9$ girls and $\frac{2}{3} \cdot 12 = 8$ boys went on the trip, and the fraction of the students on the trip who were girls is $\frac{9}{9+8} = \frac{9}{17}$. Any other size gives the same answer. With n boys and n girls, the girls and boys on the trip are in the ratio $\frac{3}{4}n : \frac{2}{3}n$, which becomes $9 : 8$ when both terms are multiplied by $\frac{12}{n}$.

Answer: (B) $\frac{9}{17}$. □

Advanced.

9. (2018 AMC 8, Problem #17) Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her bicycle toward Bella's house. They each maintain a constant speed, and Ella rides 5 times as fast as Bella walks. The distance between their houses is 2 miles, which is 10,560 feet, and Bella covers $2\frac{1}{2}$ feet with each step. How many steps will Bella take by the time she meets Ella?

(A) 704 (B) 845 (C) 1056 (D) 1760 (E) 3520

Strategy. In the same amount of time, Ella covers 5 times the distance Bella covers. Split the distance between the houses in the ratio 1 : 5.

Solution. Bella and Ella start at the same time and travel until they meet, so they travel for the same amount of time. Since Ella goes 5 times as fast, she covers 5 times the distance, and the distances covered by Bella and Ella are in the ratio 1 : 5. Together they cover the whole 10,560 feet, so Bella covers

$$\frac{1}{1+5} \cdot 10,560 = 1760 \text{ feet.}$$

At $2\frac{1}{2}$ feet per step, that is $\frac{1760}{2\frac{1}{2}} = 1760 \cdot \frac{2}{5} = 704$ steps.

Answer: (A) 704. □

10. (1999 AMC 8, Problem #22) In a far-off land three fish can be traded for two loaves of bread and a loaf of bread can be traded for four bags of rice. How many bags of rice is one fish worth?

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $2\frac{2}{3}$ (E) $3\frac{1}{3}$

Strategy. Link the two trades through the loaves of bread, as when two ratios with a shared term are combined into a three-term ratio.

Solution. One loaf is worth 4 bags of rice, so 2 loaves are worth 8 bags. Three fish are worth 2 loaves, so 3 fish are worth 8 bags of rice, and one fish is worth $\frac{8}{3} = 2\frac{2}{3}$ bags. In terms of ratios, fish to loaves is 3 : 2 and loaves to bags of rice is 1 : 4 = 2 : 8, so 3 fish, 2 loaves, and 8 bags of rice all have the same value.

Answer: (D) $2\frac{2}{3}$. □

11. (2001 AMC 8, Problem #15) Homer began peeling a pile of 44 potatoes at the rate of 3 potatoes per minute. Four minutes later Christen joined him and peeled at the rate of 5 potatoes per minute. When they finished, how many potatoes had Christen peeled?

(A) 20 (B) 24 (C) 32 (D) 33 (E) 40

Strategy. Homer works alone for the first 4 minutes. After that, the two rates add.

Solution. In the first 4 minutes Homer peels $4 \cdot 3 = 12$ potatoes, which leaves $44 - 12 = 32$. Together they peel $3 + 5 = 8$ potatoes per minute, so the remaining 32 potatoes take $\frac{32}{8} = 4$ minutes. In those 4 minutes Christen peels $4 \cdot 5 = 20$ potatoes.

Answer: (A) 20. □

12. (2008 AMC 8, Problem #20) The students in Mr. Neatkin's class took a penmanship test. Two-thirds of the boys and $\frac{3}{4}$ of the girls passed the test, and an equal number of boys and girls passed the test. What is the minimum possible number of students in the class?

(A) 12 (B) 17 (C) 24 (D) 27 (E) 36

Strategy. Set two-thirds of the boys equal to three-fourths of the girls, and find the ratio of boys to girls.

Solution. Let the class have b boys and g girls. The numbers who passed are equal, so

$$\frac{2}{3}b = \frac{3}{4}g \quad \Rightarrow \quad 8b = 9g \quad \Rightarrow \quad \frac{b}{g} = \frac{9}{8},$$

where the first step multiplies both sides by 12. So $b = 9k$ and $g = 8k$ for some positive number k . Since $k = b - g$ is a whole number, $k \geq 1$, and the class has $9k + 8k = 17k \geq 17$ students. A class of 9 boys and 8 girls works, with 6 boys and 6 girls passing.

Answer: (B) 17.

□

2. ANSWER KEY

Problem	Letter	Value
1	B	102
2	C	160
3	A	8
4	D	1920
5	B	4 : 3
6	C	100
7	B	10
8	B	$\frac{9}{17}$
9	A	704
10	D	$2\frac{2}{3}$
11	A	20
12	B	17