RATIOS, RATES, AND PROPORTIONS FOR AMC 8: PRACTICE SET SOLUTIONS

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1. Solutions

Beginner.

1. (2012 AMC 8, Problem #1) Rachelle uses 3 pounds of meat to make 8 hamburgers for her family. How many pounds of meat does she need to make 24 hamburgers for a neighborhood picnic?

(A) 6 **(B)** $6\frac{2}{3}$ **(C)** $7\frac{1}{2}$ **(D)** 8 **(E)** 9

Strategy. Going from 8 hamburgers to 24 scales the recipe. Find the scale factor and multiply the amount of meat by it.

Solution. The scale factor from 8 hamburgers to 24 hamburgers is $\frac{24}{8} = 3$. Scaling multiplies every amount in the recipe by this factor, so Rachelle needs $3 \cdot 3 = 9$ pounds of meat. Equivalently, each hamburger takes $\frac{3}{8}$ pound of meat, and $24 \cdot \frac{3}{8} = 9$.

Answer: (E) 9. □

2. (2021 AMC 10A, Problem #2) Portia's high school has 3 times as many students as Lara's high school. The two high schools have a total of 2600 students. How many students does Portia's high school have?

(A) 600 **(B)** 650 **(C)** 1950 **(D)** 2000 **(E)** 2050

Strategy. The numbers of students at the two schools are in the ratio $3 : 1$, so the total of 2600 is $3 + 1 = 4$ equal parts.

Solution. Portia's school has 3 times as many students as Lara's, so the two numbers are in the ratio $3 : 1$ and together make $3 + 1 = 4$ equal parts. Each part is $\frac{2600}{4} = 650$ students. Portia's school is 3 parts, which is $3 \cdot 650 = 1950$ students. As a check, Lara's school has 650 students, and $1950 + 650 = 2600$.

Answer: (C) 1950. □

3. (2022 AMC 10A, Problem #2) Mike cycled 15 laps in 57 minutes. Assume he cycled at a constant speed throughout. Approximately how many laps did he complete in the first 27 minutes?

(A) 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

Strategy. At a constant speed, the number of laps is proportional to the time.

Solution. Mike's rate is $\frac{15}{57} = \frac{5}{19}$ lap per minute. In 27 minutes he completes

$$27 \cdot \frac{5}{19} = \frac{135}{19} = 7\tfrac{2}{19}$$

laps, which is about 7.

Answer: (B) 7. □

4. (1986 AJHSME, Problem #1) In July 1861, 366 inches of rain fell in Cherrapunji, India. What was the average rainfall in inches per hour during that month?

(A) $\dfrac{366}{31 \times 24}$ **(B)** $\dfrac{366 \times 31}{24}$ **(C)** $\dfrac{366 \times 24}{31}$ **(D)** $\dfrac{31 \times 24}{366}$ **(E)** $366 \times 31 \times 24$

Strategy. The rainfall is a rate of 366 inches per month. Change it to inches per hour with two conversion factors, one from months to days and one from days to hours.

Solution. July is 31 days long, and a day is 24 hours. Each conversion factor is chosen so that the unit to be removed appears once on top and once on the bottom:

$$366\ \frac{\text{inches}}{\text{month}} \times \frac{1\ \text{month}}{31\ \text{days}} \times \frac{1\ \text{day}}{24\ \text{hours}} = \frac{366}{31 \times 24}\ \frac{\text{inches}}{\text{hour}}.$$

Months and days cancel, and inches per hour remain. Put another way, the month has 31×24 hours, and dividing the 366 inches by this number of hours gives the unit rate.

Answer: (A) $\dfrac{366}{31 \times 24}$. □

5. (2023 AMC 8, Problem #5) A lake contains 250 trout, along with a variety of other fish. When a marine biologist catches and releases a sample of 180 fish from the lake, 30 are identified as trout. Assume that the ratio of trout to the total number of fish is the same in both the sample and the lake. How many fish are there in the lake?

(A) 1250 **(B)** 1500 **(C)** 1750 **(D)** 1800 **(E)** 2000

Strategy. Write the ratio of trout to all fish for the sample and for the lake, in the same order, and set the two ratios equal.

Solution. Let N be the number of fish in the lake. Both fractions below are trout over all fish:

$$\frac{30}{180} = \frac{250}{N}.$$

By cross-multiplication, $30N = 180 \cdot 250 = 45{,}000$, so $N = 1500$. Equivalently, $\frac{30}{180} = \frac{1}{6}$, so one fish in six is a trout, and the lake holds $6 \cdot 250 = 1500$ fish.

Answer: (B) 1500. □

Intermediate.

6. (2019 AMC 10B, Problem #1) Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the smaller container to the volume of the larger container?

(A) $\dfrac{5}{8}$ **(B)** $\dfrac{4}{5}$ **(C)** $\dfrac{7}{8}$ **(D)** $\dfrac{9}{10}$ **(E)** $\dfrac{11}{12}$

Strategy. The same amount of water fills $\frac{5}{6}$ of the first container and $\frac{3}{4}$ of the second. Set these equal.

Solution. Let the volumes be V_1 and V_2. Then $\frac{5}{6}V_1 = \frac{3}{4}V_2$, so

$$\frac{V_1}{V_2} = \frac{3/4}{5/6} = \frac{3}{4} \cdot \frac{6}{5} = \frac{9}{10}.$$

This ratio is less than 1, so the first container is the smaller one, and the requested ratio is $\frac{9}{10}$.

Answer: (D) $\frac{9}{10}$. □

7. (2020 AMC 10B, Problem #3) The ratio of w to x is $4 : 3$, the ratio of y to z is $3 : 2$, and the ratio of z to x is $1 : 6$. What is the ratio of w to y?

(A) $4 : 3$ **(B)** $3 : 2$ **(C)** $8 : 3$ **(D)** $4 : 1$ **(E)** $16 : 3$

Strategy. Chain the ratios so that the intermediate letters cancel: $\frac{w}{y} = \frac{w}{x} \cdot \frac{x}{z} \cdot \frac{z}{y}$.

Solution. From the given ratios, $\frac{w}{x} = \frac{4}{3}$, $\frac{x}{z} = 6$, and $\frac{z}{y} = \frac{2}{3}$. Therefore

$$\frac{w}{y} = \frac{w}{x} \cdot \frac{x}{z} \cdot \frac{z}{y} = \frac{4}{3} \cdot 6 \cdot \frac{2}{3} = \frac{16}{3},$$

so the ratio of w to y is $16 : 3$.

Answer: (E) $16 : 3$. □

8. (2014 AMC 10A, Problem #6) Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

(A) $\frac{bde}{ac}$ **(B)** $\frac{ac}{bde}$ **(C)** $\frac{abde}{c}$ **(D)** $\frac{bcde}{a}$ **(E)** $\frac{abc}{de}$

Strategy. Change one quantity at a time. Changing the number of cows and changing the number of days each multiply the amount of milk by a scale factor.

Solution. First change only the number of cows. In c days, d cows give $\frac{d}{a}$ times as much milk as a cows, which is $b \cdot \frac{d}{a} = \frac{bd}{a}$ gallons. Next change only the number of days. In e days, these d cows give $\frac{e}{c}$ times as much milk as in c days, which is

$$\frac{bd}{a} \cdot \frac{e}{c} = \frac{bde}{ac}$$

gallons. Equivalently, the unit rate is $\frac{b}{ac}$ gallon for one cow in one day, and d cows for e days give $de \cdot \frac{b}{ac} = \frac{bde}{ac}$ gallons.

Answer: (A) $\frac{bde}{ac}$. □

9. (2021 AMC 10A, Problem #3) The sum of two natural numbers is 17,402. One of the two numbers is divisible by 10. If the units digit of that number is erased, the other number is obtained. What is the difference of these two numbers?

(A) 10,272 **(B)** 11,700 **(C)** 13,362 **(D)** 14,238 **(E)** 15,426

Strategy. The units digit of a multiple of 10 is 0, so erasing it divides the number by 10.

Solution. Let x be the number obtained by erasing. The number divisible by 10 has units digit 0, and erasing that 0 divides it by 10. So that number is $10x$. Then $x + 10x = 11x = 17{,}402$, so $x = 1582$. The difference is

$$10x - x = 9x = 9 \cdot 1582 = 14{,}238.$$

Answer: (D) 14,238. □

10. (2020 AMC 10A, Problem #4) A driver travels for 2 hours at 60 miles per hour, during which her car gets 30 miles per gallon of gasoline. She is paid \$0.50 per mile, and her only expense is gasoline at \$2.00 per gallon. What is her net rate of pay, in dollars per hour, after this expense?

(A) 20 **(B)** 22 **(C)** 24 **(D)** 25 **(E)** 26

Strategy. Find the pay and the cost for the whole trip, then divide the difference by the number of hours.

Solution. In 2 hours at 60 miles per hour, the trip covers $2 \cdot 60 = 120$ miles, and the pay is $120 \cdot \$0.50 = \$60$. At 30 miles per gallon, the 120 miles take $\frac{120}{30} = 4$ gallons, which cost $4 \cdot \$2.00 = \$8$. After this expense, $\$60 - \$8 = \$52$ remains for the 2 hours, so the net rate of pay is $\frac{52}{2} = 26$ dollars per hour.

Answer: (E) 26. □

Advanced.

11. (2004 AMC 10B, Problem #7) On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

Strategy. The exchange turned d U.S. dollars into $d + 60$ Canadian dollars. These two amounts are in the ratio $7 : 10$, which gives a proportion for d.

Solution. Isabella spent 60 Canadian dollars and had d left, so the exchange gave her $d+60$ Canadian dollars. She received 10 Canadian dollars for every 7 U.S. dollars, so her U.S. and Canadian amounts are in the ratio $7 : 10$. Both fractions below are U.S. dollars over Canadian dollars:

$$\frac{d}{d+60} = \frac{7}{10}.$$

By cross-multiplication, $10d = 7(d + 60) = 7d + 420$, so $3d = 420$ and $d = 140$. The sum of the digits of 140 is $1 + 4 + 0 = 5$.

In terms of parts, d is 7 parts and $d + 60$ is 10 parts, so their difference, 60, is 3 parts of 20 dollars each, and $d = 7 \cdot 20 = 140$. As a check, 140 U.S. dollars buy $\frac{10}{7} \cdot 140 = 200$ Canadian dollars, and $200 - 60 = 140$.

Answer: (A) 5. □

12. (2019 AMC 10B, Problem #11) Two jars each contain the same number of marbles, and every marble is either blue or green. In Jar 1 the ratio of blue to green marbles is $9 : 1$, and the ratio of blue to green marbles in Jar 2 is $8 : 1$. There are 95 green marbles in all. How many more blue marbles are in Jar 1 than in Jar 2?

(A) 5 **(B)** 10 **(C)** 25 **(D)** 45 **(E)** 50

Strategy. A jar is 10 parts in the ratio $9 : 1$ but 9 parts in the ratio $8 : 1$, so the parts of the two jars have different sizes. Scale both ratios so that each jar is 90 parts.

Solution. Jar 1 is $9 + 1 = 10$ equal parts and Jar 2 is $8 + 1 = 9$ equal parts. The jars hold the same number of marbles, so a part of Jar 1 is smaller than a part of Jar 2, and the two ratios cannot be combined yet. The least common multiple of 10 and 9 is 90, so scale both ratios to make each jar 90 parts:

$$\text{Jar 1: } 9 : 1 = 81 : 9, \qquad \text{Jar 2: } 8 : 1 = 80 : 10.$$

Now every part has the same size, say k marbles. The green marbles make $9 + 10 = 19$ parts, so $19k = 95$ and $k = 5$. Jar 1 has 81 parts of blue marbles and Jar 2 has 80, so Jar 1 has 1 part, or 5, more blue marbles. As a check, each jar holds $90 \cdot 5 = 450$ marbles: 405 blue and 45 green in Jar 1, and 400 blue and 50 green in Jar 2.

Answer: (A) 5. □

13. (1985 AHSME, Problem #4) A large bag of coins contains pennies, dimes and quarters. There are twice as many dimes as pennies and three times as many quarters as dimes. An amount of money which could be in the bag is

(A) \$306 **(B)** \$333 **(C)** \$342 **(D)** \$348 **(E)** \$360

Strategy. Join the two ratios into one three-term ratio of pennies to dimes to quarters. The coins then come in equal groups of 1 penny, 2 dimes, and 6 quarters, so the amount of money is a whole number of times the value of one group.

Solution. Pennies to dimes is $1:2$ and dimes to quarters is $1:3$. The dimes are 2 parts in the first ratio and 1 part in the second, so scale the second ratio to $2:6$. Then pennies to dimes to quarters is $1:2:6$. With k pennies, where k is a whole number, the bag has $2k$ dimes and $6k$ quarters, worth

$$k \cdot 1 + 2k \cdot 10 + 6k \cdot 25 = k + 20k + 150k = 171k$$

cents in all, which is k times \$1.71. For $k = 200$ the amount is \$342: 200 pennies, 400 dimes, and 1200 quarters are worth $\$2 + \$40 + \$300 = \$342$. None of the other choices is a whole-number multiple of \$1.71, because each lies strictly between two consecutive multiples:

$$178 \cdot \$1.71 = \$304.38 < \$306 < \$306.09 = 179 \cdot \$1.71,$$
$$194 \cdot \$1.71 = \$331.74 < \$333 < \$333.45 = 195 \cdot \$1.71,$$
$$203 \cdot \$1.71 = \$347.13 < \$348 < \$348.84 = 204 \cdot \$1.71,$$
$$210 \cdot \$1.71 = \$359.10 < \$360 < \$360.81 = 211 \cdot \$1.71.$$

Answer: (C) \$342. □

14. (2015 AMC 8, Problem #17) Jeremy's father drives him to school in rush hour traffic in 20 minutes. One day there is no traffic, so his father can drive him 18 miles per hour faster and gets him to school in 12 minutes. How far in miles is it to school?

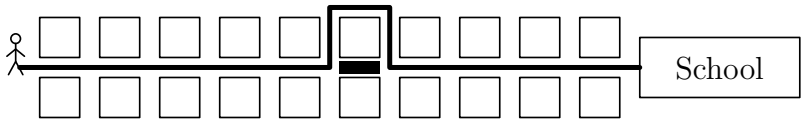
(A) 4 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

Strategy. Both drives cover the same distance, and the faster one takes $\frac{12}{20} = \frac{3}{5}$ of the time, so the speeds are in the ratio $3:5$. Their difference, 18 miles per hour, is $5 - 3 = 2$ parts.

Solution. Without traffic the drive takes $\frac{12}{20} = \frac{3}{5}$ of the time it takes in traffic. Covering the same distance in $\frac{3}{5}$ of the time takes $\frac{5}{3}$ times the speed, so the speeds in traffic and without traffic are in the ratio $3:5$. Write them as $3k$ and $5k$ miles per hour. Their difference is $5k - 3k = 2k = 18$, so $k = 9$, and the speed in traffic is $3 \cdot 9 = 27$ miles per hour. In 20 minutes, which is $\frac{1}{3}$ hour, the father drives $27 \cdot \frac{1}{3} = 9$ miles. As a check, without traffic he drives $5 \cdot 9 = 45$ miles per hour for 12 minutes, which is $\frac{1}{5}$ hour, and $45 \cdot \frac{1}{5} = 9$ miles as well.

Answer: (D) 9. □

15. (2023 AMC 8, Problem #15) Viswam walks half a mile to get to school each day. His route consists of 10 city blocks of equal length and he takes one minute to walk each block. Today, after walking 5 blocks, Viswam discovers that he has to make a detour, walking 3 blocks of equal length instead of 1 block to reach the next corner. From the time he starts his detour, at what speed, in miles per hour, must Viswam walk in order to arrive at school at his usual time?



(A) 4 **(B)** 4.2 **(C)** 4.5 **(D)** 4.8 **(E)** 5

Strategy. Count the blocks and the minutes left when the detour starts. Then change the speed from blocks per minute to miles per hour with conversion factors.

Solution. The usual walk is 10 blocks at 1 minute per block, so it takes 10 minutes. When the detour starts, Viswam has walked 5 blocks in 5 minutes, so 5 minutes are left. Of the

5 blocks left on his route, the detour replaces 1 with 3 blocks of the same length, so he still has $5 - 1 + 3 = 7$ blocks to walk in those 5 minutes. Ten blocks make half a mile, so 20 blocks make 1 mile. Therefore his speed must be

$$\frac{7 \text{ blocks}}{5 \text{ minutes}} \times \frac{1 \text{ mile}}{20 \text{ blocks}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{7 \cdot 60}{5 \cdot 20} \frac{\text{miles}}{\text{hour}} = 4.2 \frac{\text{miles}}{\text{hour}}.$$

Answer: (B) 4.2. □

2. Answer Key

Problem	Letter	Value
1	E	9
2	C	1950
3	B	7
4	A	$\frac{366}{31\times24}$
5	B	1500
6	D	$\frac{9}{10}$
7	E	16 : 3
8	A	$\frac{bde}{ac}$
9	D	14,238
10	E	26
11	A	5
12	A	5
13	C	$342
14	D	9
15	B	4.2