TRIGONOMETRIC IDENTITIES AND EQUATIONS FOR AMC 12: SOLUTIONS

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1. Solutions

Beginner.

1. (1983 AHSME, Problem #11) Simplify $\sin(x-y)\cos y + \cos(x-y)\sin y$.

(A) 1 **(B)** $\sin x$ **(C)** $\cos x$ **(D)** $\sin x \cos 2y$ **(E)** $\cos x \cos 2y$

Strategy. The expression has the form $\sin\alpha\cos\beta + \cos\alpha\sin\beta$, with $\alpha = x-y$ and $\beta = y$.

Solution. By the sum formula for sine with $\alpha = x-y$ and $\beta = y$, the expression equals $\sin\big((x-y)+y\big) = \sin x$.

Answer: (B) $\sin x$. □

2. (2018 AMC 12A, Problem #9) Which of the following describes the largest subset of values of y within the closed interval $[0,\pi]$ for which

$$\sin(x+y) \le \sin(x) + \sin(y)$$

for every x between 0 and π, inclusive?

(A) $y = 0$ **(B)** $0 \le y \le \frac{\pi}{4}$ **(C)** $0 \le y \le \frac{\pi}{2}$ **(D)** $0 \le y \le \frac{3\pi}{4}$ **(E)** $0 \le y \le \pi$

Strategy. Expand $\sin(x+y)$ and compare each of its two products with the matching term of $\sin x + \sin y$.

Solution. For x and y in $[0,\pi]$, $\sin x \ge 0$ and $\sin y \ge 0$, while $\cos x \le 1$ and $\cos y \le 1$. Hence $\sin x \cos y \le \sin x$ and $\cos x \sin y \le \sin y$, and by the sum formula

$$\sin(x+y) = \sin x\cos y + \cos x \sin y \le \sin x + \sin y.$$

So the inequality holds for every x in $[0,\pi]$ whenever y is in $[0,\pi]$, and the largest subset is the whole interval $0 \le y \le \pi$.

Answer: (E) $0 \le y \le \pi$. □

3. (1973 AHSME, Problem #17) If θ is an acute angle and

$$\sin\frac{1}{2}\theta = \sqrt{\frac{x-1}{2x}},$$

then $\tan\theta$ equals

(A) x **(B)** $\dfrac{1}{x}$ **(C)** $\dfrac{\sqrt{x-1}}{x+1}$ **(D)** $\dfrac{\sqrt{x^2-1}}{x}$ **(E)** $\sqrt{x^2-1}$

Strategy. The half-angle formula relates $\sin^2\frac{\theta}{2}$ to $\cos\theta$. Find $\cos\theta$, then $\sin\theta$, then their quotient.

Solution. By the half-angle formula, $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$. Squaring the given equation gives $\frac{x-1}{2x} = \frac{1-\cos\theta}{2}$, so $1-\cos\theta = \frac{x-1}{x}$ and $\cos\theta = \frac{1}{x}$. Since θ is acute, $\cos\theta > 0$, so $x > 0$, and

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{x^2}} = \frac{\sqrt{x^2-1}}{x}.$$

Therefore $\tan\theta = \frac{\sin\theta}{\cos\theta} = \sqrt{x^2-1}$.

Answer: (E) $\sqrt{x^2-1}$. □

4. (2025 AMC 12B, Problem #16) An analog clock starts at midnight and runs for 2025 minutes before stopping. What is the tangent of the acute angle between the hour hand and the minute hand when the clock stops?

(A) 0 **(B)** $\sqrt{2}-1$ **(C)** $2-\sqrt{2}$ **(D)** $\dfrac{\sqrt{2}}{2}$ **(E)** $3-\sqrt{2}$

Strategy. Find the time the clock shows when it stops and the positions of the two hands. The tangent of the angle between them comes from the half-angle formula.

Solution. Since 2025 minutes is 33 hours 45 minutes, and the hands return to the same positions every 12 hours, the clock stops showing 9:45. Measure the positions of the hands clockwise from the 12. The minute hand, at 45 minutes, is at $\frac{45}{60} \cdot 360^\circ = 270^\circ$. The hour hand turns 30° per hour, so at 9:45 it is at $9.75 \cdot 30^\circ = 292.5^\circ$. The angle between the hands is $292.5^\circ - 270^\circ = 22.5^\circ$, which is acute. By the half-angle formula with $\theta = 45^\circ$,

$$\tan 22.5^\circ = \frac{1 - \cos 45^\circ}{\sin 45^\circ} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1.$$

Answer: (B) $\sqrt{2} - 1$. □

5. (2010 AMC 12B, Problem #13) In $\triangle ABC$, $\cos(2A - B) + \sin(A + B) = 2$ and $AB = 4$. What is BC?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** $2\sqrt{3}$

Strategy. A cosine and a sine are each at most 1, so their sum is 2 only when both equal 1.

Solution. Since $\cos(2A - B) \le 1$ and $\sin(A + B) \le 1$, the sum is 2 only if both terms equal 1. As $0^\circ < A + B < 180^\circ$, the equation $\sin(A + B) = 1$ gives $A + B = 90^\circ$. Then A and B each lie between 0° and 90°, so $-90^\circ < 2A - B < 180^\circ$, and the only angle in this range with cosine 1 is 0°. Hence $B = 2A$, so $3A = 90^\circ$, $A = 30^\circ$, $B = 60^\circ$, and $C = 90^\circ$. The side $AB = 4$ is the hypotenuse, opposite the right angle, and BC is opposite the 30° angle, so $BC = AB \sin 30^\circ = 2$.

Answer: (C) 2. □

Intermediate.

6. (2024 AMC 12B, Problem #21) The measures of the smallest angles of three different right triangles sum to 90°. All three triangles have side lengths that are primitive Pythagorean triples. Two of them are 3-4-5 and 5-12-13. What is the perimeter of the third triangle?

(A) 40 **(B)** 126 **(C)** 154 **(D)** 176 **(E)** 208

Strategy. The smallest angle of the third triangle is 90° minus the sum of the other two smallest angles. Its tangent comes from the tangent of that sum.

Solution. The smallest angle of a right triangle is the acute angle opposite the shorter leg, and its tangent is the shorter leg divided by the longer leg. So the smallest angles α of the 3-4-5 triangle and β of the 5-12-13 triangle have $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{5}{12}$. Their product $\frac{15}{48}$ is not 1, so

$$\tan(\alpha + \beta) = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{14}{12}}{\frac{33}{48}} = \frac{56}{33}.$$

The smallest angle γ of the third triangle is $90^\circ - (\alpha + \beta)$, so by the complementary-angle identity $\tan\gamma = \frac{1}{\tan(\alpha+\beta)} = \frac{33}{56}$. Thus the legs $p < q$ of the third triangle satisfy $\frac{p}{q} = \frac{33}{56}$, a fraction in lowest terms. In a primitive triple the legs have no common factor, because a prime dividing both legs would divide the square of the hypotenuse and hence the hypotenuse. So $p = 33$ and $q = 56$, the hypotenuse is

$$\sqrt{33^2 + 56^2} = \sqrt{1089 + 3136} = \sqrt{4225} = 65,$$

and the perimeter is $33 + 56 + 65 = 154$.

Answer: (C) 154. □

7. (2004 AMC 12A, Problem #21) If

$$\sum_{n=0}^{\infty} \cos^{2n}\theta = 5,$$

what is the value of $\cos 2\theta$?

(A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{\sqrt{5}}{5}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{4}{5}$

Strategy. Each term of the series is $\cos^2\theta$ times the term before it. The value of the sum fixes $\sin^2\theta$, and the form of the double-angle formula that contains only $\sin^2\theta$ gives $\cos 2\theta$.

Solution. The terms $1, \cos^2\theta, \cos^4\theta, \ldots$ form a geometric sequence with common ratio $\cos^2\theta$. If $\cos^2\theta = 1$, every term is 1 and the series has no finite sum, so $0 \le \cos^2\theta < 1$ and the sum is

$$\frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}.$$

Setting this equal to 5 gives $\sin^2\theta = \frac{1}{5}$. By the double-angle formula,

$$\cos 2\theta = 1 - 2\sin^2\theta = 1 - \frac{2}{5} = \frac{3}{5}.$$

Answer: (D) $\frac{3}{5}$. □

8. (1975 AHSME, Problem #30) Let $x = \cos 36^\circ - \cos 72^\circ$. Then x equals

(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $3-\sqrt{6}$ **(D)** $2\sqrt{3}-3$ **(E)** none of these

Strategy. The angle 72° is twice 36°, and 36° is twice 18°, where $\sin 18^\circ = \cos 72^\circ$. The double-angle formula, used in two forms, gives two equations between $\cos 36^\circ$ and $\cos 72^\circ$.

Solution. Let $a = \cos 36^\circ$ and $b = \cos 72^\circ$. By the double-angle formula, $b = \cos 72^\circ = 2\cos^2 36^\circ - 1 = 2a^2 - 1$. Also $\sin 18^\circ = \cos 72^\circ = b$ by the complementary-angle identity, so $a = \cos 36^\circ = 1 - 2\sin^2 18^\circ = 1 - 2b^2$. Adding the two equations gives

$$a + b = 2a^2 - 2b^2 = 2(a-b)(a+b).$$

Both a and b are cosines of acute angles, so $a + b > 0$. Dividing by $a + b$ gives $a - b = \frac{1}{2}$, so $x = \frac{1}{2}$.

Answer: (B) $\frac{1}{2}$. □

9. (1974 AHSME, Problem #22) The minimum value of $\sin\frac{A}{2} - \sqrt{3}\cos\frac{A}{2}$ is attained when A is

(A) -180° **(B)** 60° **(C)** 120° **(D)** 0° **(E)** none of these

Strategy. Write the expression in amplitude-phase form and find where its sine equals -1.

Solution. Here $a = 1$ and $b = -\sqrt{3}$, so $R = \sqrt{1+3} = 2$, $\cos\varphi = \frac{1}{2}$, and $\sin\varphi = -\frac{\sqrt{3}}{2}$; take $\varphi = -60^\circ$. In amplitude-phase form,

$$\sin\frac{A}{2} - \sqrt{3}\cos\frac{A}{2} = 2\sin\Big(\frac{A}{2} - 60^\circ\Big).$$

Its minimum value -2 is attained exactly when $\frac{A}{2} - 60^\circ = -90^\circ + 360^\circ k$, that is, when $A = -60^\circ + 720^\circ k$ for an integer k. For the listed angles -180°, 60°, 120°, and 0°, the number $A + 60^\circ$ is -120°, 120°, 180°, or 60°, never a multiple of 720°. (At these angles the expression equals -1, -1, 0, and $-\sqrt{3}$.) So none of the listed angles gives the minimum.

Answer: (E) none of these. □

10. (2003 AMC 12B, Problem #23) The number of x-intercepts on the graph of $y = \sin(\frac{1}{x})$ in the interval $(0.0001, 0.001)$ is closest to

(A) 2900 **(B)** 3000 **(C)** 3100 **(D)** 3200 **(E)** 3300

Strategy. The graph meets the x-axis where $\frac{1}{x}$ is a multiple of π. Count the multiples of π between 1000 and 10000.

Solution. As x runs over $(0.0001, 0.001)$, the number $u = \frac{1}{x}$ runs over $(1000, 10000)$, each value once. Since $\sin u = 0$ exactly when $u = k\pi$ for an integer k, the number of x-intercepts is the number of integers k with $1000 < k\pi < 10000$. Since $3.1415 < \pi < 3.1416$,

$$318\pi < 999.1 < 1000 < 1002.1 < 319\pi,$$

$$3183\pi < 9999.8 < 10000 < 10002.5 < 3184\pi,$$

so k runs from 319 to 3183, which gives $3183 - 319 + 1 = 2865$ intercepts. Of the listed numbers, 2900 is closest.

Answer: (A) 2900. □

11. (1996 AIME, Problem #10) Find the smallest positive integer solution to

$$\tan(19x^\circ) = \frac{\cos 96^\circ + \sin 96^\circ}{\cos 96^\circ - \sin 96^\circ}.$$

Strategy. Divide the numerator and the denominator of the right side by $\cos 96^\circ$. Since $\tan 45^\circ = 1$, the result is the tangent of a sum. Then solve an equation between two tangents.

Solution. Since $\cos 96^\circ \neq 0$, dividing the numerator and the denominator by $\cos 96^\circ$ gives

$$\frac{1 + \tan 96^\circ}{1 - \tan 96^\circ} = \frac{\tan 45^\circ + \tan 96^\circ}{1 - \tan 45^\circ \tan 96^\circ} = \tan(45^\circ + 96^\circ) = \tan 141^\circ,$$

where the formula for the tangent of a sum applies because $\tan 45^\circ \tan 96^\circ = \tan 96^\circ$ is negative and so is not 1. By the theorem on equal tangents, $\tan(19x^\circ) = \tan 141^\circ$ exactly when $19x = 141 + 180k$ for an integer k, that is, $19x \equiv 141 \pmod{180}$. Since $19 \cdot 19 = 361 = 2 \cdot 180 + 1$, multiplying by 19 gives $x \equiv 19 \cdot 141 = 2679 = 14 \cdot 180 + 159 \pmod{180}$. So the positive integer solutions are 159, 339, 519, and so on, and the smallest is 159. As a check, $19 \cdot 159 = 3021 = 141 + 16 \cdot 180$.

Answer: 159. □

Advanced.

12. (1999 AHSME, Problem #27) In triangle ABC, $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Then $\angle C$ in degrees is

(A) 30 **(B)** 60 **(C)** 90 **(D)** 120 **(E)** 150

Strategy. In the sum of the squares of the two left sides, the squared terms combine by the Pythagorean identity, and the cross terms form a multiple of $\sin(A + B)$, which equals $\sin C$. Two values of C have that sine; check which one the equations allow.

Solution. Squaring the equations and adding gives $(3\sin A + 4\cos B)^2 + (4\sin B + 3\cos A)^2 = 36 + 1 = 37$. The left side is

$$9(\sin^2 A + \cos^2 A) + 16(\cos^2 B + \sin^2 B) + 24(\sin A \cos B + \cos A \sin B)$$
$$= 25 + 24\sin(A + B),$$

so $\sin(A+B)=\frac{1}{2}$. Since $A+B=180^\circ-C$, $\sin C=\sin(A+B)=\frac{1}{2}$, and C is 30° or 150°. If C were 150°, then $A<30^\circ$, so $\sin A<\frac{1}{2}$ because sine increases from 0° to 90°; then $3\sin A+4\cos B<\frac{3}{2}+4<6$, which contradicts the first equation. Hence $C=30^\circ$.

Answer: (A) 30. □

13. (2012 AIME II, Problem #9) Let x and y be real numbers such that $\frac{\sin x}{\sin y}=3$ and $\frac{\cos x}{\cos y}=\frac{1}{2}$. The value of $\frac{\sin 2x}{\sin 2y}+\frac{\cos 2x}{\cos 2y}$ can be expressed in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p+q$.

Strategy. The double-angle formula for sine makes the first quotient the product of the two given quotients. For the second quotient, find $\cos^2 x$ and $\cos^2 y$ from the given ratios and the Pythagorean identity.

Solution. The given ratios say $\sin x=3\sin y$ and $\cos x=\frac{1}{2}\cos y$, with $\sin y\neq 0$ and $\cos y\neq 0$. By the double-angle formula,

$$\frac{\sin 2x}{\sin 2y}=\frac{2\sin x\cos x}{2\sin y\cos y}=3\cdot\frac{1}{2}=\frac{3}{2}.$$

Next, $\sin^2 x+\cos^2 x=1$ gives $9\sin^2 y+\frac{1}{4}\cos^2 y=1$, that is, $9(1-\cos^2 y)+\frac{1}{4}\cos^2 y=1$. So $\frac{35}{4}\cos^2 y=8$, $\cos^2 y=\frac{32}{35}$, and $\cos^2 x=\frac{1}{4}\cos^2 y=\frac{8}{35}$. By the double-angle formula,

$$\cos 2x=2\cos^2 x-1=\frac{16}{35}-1=-\frac{19}{35},\qquad \cos 2y=2\cos^2 y-1=\frac{64}{35}-1=\frac{29}{35},$$

so $\frac{\cos 2x}{\cos 2y}=-\frac{19}{29}$. The required value is $\frac{3}{2}-\frac{19}{29}=\frac{87-38}{58}=\frac{49}{58}$, which is in lowest terms. Hence $p+q=49+58=107$.

Answer: 107. □

14. (1997 AIME, Problem #11) Let

$$x=\frac{\displaystyle\sum_{n=1}^{44}\cos n^\circ}{\displaystyle\sum_{n=1}^{44}\sin n^\circ}.$$

What is the greatest integer that does not exceed $100x$?

Strategy. Pair n° with $(45-n)^\circ$. By the sum-to-product formulas, each pair of cosines and the matching pair of sines share the factor $\cos(n-22.5)^\circ$, so every pair has the same ratio.

Solution. The angles $1^\circ, 2^\circ, \ldots, 44^\circ$ form the 22 pairs $\{n^\circ, (45-n)^\circ\}$ with $n=1,2,\ldots,22$. For each pair, the sum-to-product formulas give

$$\cos n^\circ+\cos(45-n)^\circ=2\cos 22.5^\circ\cos(n-22.5)^\circ,$$
$$\sin n^\circ+\sin(45-n)^\circ=2\sin 22.5^\circ\cos(n-22.5)^\circ.$$

Let c be the sum of $\cos(n-22.5)^\circ$ over $n=1,2,\ldots,22$; each term is positive, so $c>0$. Adding over the pairs shows that the numerator of x is $2c\cos 22.5^\circ$ and the denominator is $2c\sin 22.5^\circ$, so $x=\frac{1}{\tan 22.5^\circ}$. By the half-angle formula with $\theta=45^\circ$,

$$\tan 22.5^\circ=\frac{\sin 45^\circ}{1+\cos 45^\circ}=\frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}=\frac{\sqrt{2}}{2+\sqrt{2}}=\frac{\sqrt{2}(2-\sqrt{2})}{2}=\sqrt{2}-1,$$

so $x = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$. Since $1.41^2 = 1.9881 < 2 < 2.0164 = 1.42^2$, it follows that $141 < 100\sqrt{2} < 142$, so $241 < 100x < 242$. The greatest integer not exceeding $100x$ is 241.

Answer: 241. □

15. (2021 AMC 12A, Problem #19) How many solutions does the equation

$$\sin\left(\frac{\pi}{2}\cos x\right) = \cos\left(\frac{\pi}{2}\sin x\right)$$

have in the closed interval $[0, \pi]$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Strategy. Write the right side as a sine, using $\cos t = \sin\left(\frac{\pi}{2} - t\right)$, and apply the theorem on equal sines. Each case becomes an equation in $\sin x$ and $\cos x$ that the amplitude-phase form solves.

Solution. Since $\cos\left(\frac{\pi}{2}\sin x\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\sin x\right)$, the theorem on equal sines says the equation holds exactly when, for some integer k,

$$\frac{\pi}{2}\cos x = \frac{\pi}{2} - \frac{\pi}{2}\sin x + 2k\pi \qquad \text{or} \qquad \frac{\pi}{2}\cos x = \pi - \left(\frac{\pi}{2} - \frac{\pi}{2}\sin x\right) + 2k\pi,$$

that is, $\sin x + \cos x = 1 + 4k$ or $\cos x - \sin x = 1 + 4k$. In amplitude-phase form (with $a = b = 1$, so $R = \sqrt{2}$ and $\varphi = \frac{\pi}{4}$; and with $a = -1$, $b = 1$, so $R = \sqrt{2}$ and $\varphi = \frac{3\pi}{4}$), $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ and $\cos x - \sin x = \sqrt{2}\sin\left(x + \frac{3\pi}{4}\right)$, so both lie between $-\sqrt{2}$ and $\sqrt{2}$. The only number of the form $1 + 4k$ in that range is 1. So the equation holds exactly when $\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ or $\sin\left(x + \frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

For x in $[0, \pi]$, the angle $x + \frac{\pi}{4}$ lies in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$, where its sine is $\frac{\sqrt{2}}{2}$ only at $\frac{\pi}{4}$ and $\frac{3\pi}{4}$; this gives $x = 0$ and $x = \frac{\pi}{2}$. The angle $x + \frac{3\pi}{4}$ lies in $\left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$, where its sine is $\frac{\sqrt{2}}{2}$ only at $\frac{3\pi}{4}$; this gives $x = 0$ again. So the solutions are $x = 0$ and $x = \frac{\pi}{2}$: two solutions.

Answer: (C) 2. □

16. (2007 AMC 12A, Problem #24) For each integer $n > 1$, let $F(n)$ be the number of solutions of the equation $\sin x = \sin nx$ on the interval $[0, \pi]$. What is $\displaystyle\sum_{n=2}^{2007} F(n)$?

(A) 2,014,524 **(B)** 2,015,028 **(C)** 2,015,033 **(D)** 2,016,532 **(E)** 2,017,033

Strategy. Factor $\sin nx - \sin x$ with a sum-to-product formula. Count the zeros of each factor in $[0, \pi]$, and find the solutions the two factors share.

Solution. By the sum-to-product formula,

$$\sin nx - \sin x = 2\cos\frac{(n+1)x}{2}\sin\frac{(n-1)x}{2},$$

so $\sin x = \sin nx$ exactly when one of the two factors is 0.

- The factor $\sin\frac{(n-1)x}{2}$ is 0 exactly when $x = \frac{2k\pi}{n-1}$ for an integer k. These numbers lie in $[0, \pi]$ for $k = 0, 1, \ldots, \left\lfloor\frac{n-1}{2}\right\rfloor$, which gives $\left\lfloor\frac{n-1}{2}\right\rfloor + 1$ solutions.
- The factor $\cos\frac{(n+1)x}{2}$ is 0 exactly when $x = \frac{(2j+1)\pi}{n+1}$ for an integer j. These lie in $[0, \pi]$ for $j = 0, 1, \ldots, \left\lfloor\frac{n}{2}\right\rfloor$, which gives $\left\lfloor\frac{n}{2}\right\rfloor + 1$ solutions.

Since $\left\lfloor\frac{n-1}{2}\right\rfloor + \left\lfloor\frac{n}{2}\right\rfloor = n - 1$ (for even n the two terms are $\frac{n-2}{2}$ and $\frac{n}{2}$, and for odd n both are $\frac{n-1}{2}$), the two lists have $n + 1$ entries in all. A solution in both lists satisfies $\frac{(n-1)x}{2} = k\pi$ and $\frac{(n+1)x}{2} = \frac{\pi}{2} + j\pi$; subtracting the first equation from the second gives $x = \frac{\pi}{2} + (j - k)\pi$,

and the only such number in $[0, \pi]$ is $\frac{\pi}{2}$. The number $\frac{\pi}{2}$ is in the first list exactly when $\frac{n-1}{4}$ is an integer, and then it is also in the second list, since $\frac{(n+1)\pi}{4} = \frac{\pi}{2} + \frac{n-1}{4}\pi$. Hence $F(n) = n$ when $n \equiv 1 \pmod 4$, and $F(n) = n + 1$ otherwise. Summing $n + 1$ from $n = 2$ to 2007 gives

$$\sum_{n=2}^{2007}(n+1) = 3 + 4 + \cdots + 2008 = \frac{2008 \cdot 2009}{2} - 3 = 2{,}017{,}033,$$

and the values $n \equiv 1 \pmod 4$ from 2 to 2007 are $5, 9, \ldots, 2005$, which are $\frac{2005-5}{4} + 1 = 501$ numbers. Therefore $\sum_{n=2}^{2007} F(n) = 2{,}017{,}033 - 501 = 2{,}016{,}532$.

Answer: (D) 2,016,532. □

2. Answer Key

Problem	Letter	Value
1	B	$\sin x$
2	E	$0 \leq y \leq \pi$
3	E	$\sqrt{x^2-1}$
4	B	$\sqrt{2}-1$
5	C	2
6	C	154
7	D	$\frac{3}{5}$
8	B	$\frac{1}{2}$
9	E	none of these
10	A	2900
11		159
12	A	30
13		107
14		241
15	C	2
16	D	2,016,532