TRIGONOMETRIC IDENTITIES AND EQUATIONS FOR AMC 12: PRACTICE SET

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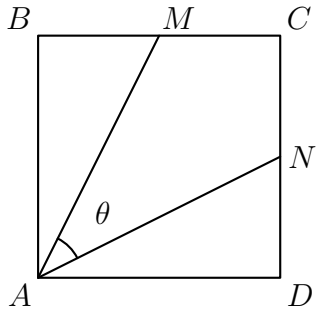
1. Problems

Beginner.

1. (2020 AMC 12B, Problem #7) Two nonhorizontal, non-vertical lines in the xy-coordinate plane intersect to form a $45°$ angle. One line has slope equal to 6 times the slope of the other line. What is the greatest possible value of the product of the slopes of the two lines?

(A) $\frac{1}{6}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** 3 **(E)** 6

2. (1987 AHSME, Problem #14) $ABCD$ is a square and M and N are the midpoints of BC and CD, respectively. Then $\sin\theta =$

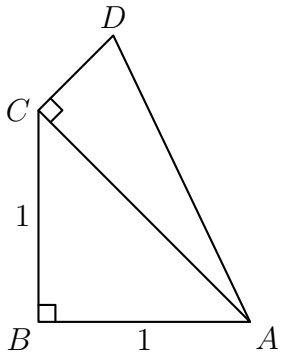


(A) $\frac{\sqrt{5}}{5}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{\sqrt{10}}{5}$ **(D)** $\frac{4}{5}$ **(E)** none of these

3. (1996 AHSME, Problem #25) Given that $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value that $3x + 4y$ can have?

(A) 72 **(B)** 73 **(C)** 74 **(D)** 75 **(E)** 76

4. (2019 AMC 12B, Problem #12) Right triangle ACD with right angle at C is constructed outwards on the hypotenuse $\overline{AC}$ of isosceles right triangle ABC with leg length 1, as shown, so that the two triangles have equal perimeters. What is $\sin(2\angle BAD)$?



(A) $\frac{1}{3}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{7}{9}$ **(E)** $\frac{\sqrt{3}}{2}$

5. (1999 AHSME, Problem #18) How many zeros does $f(x) = \cos(\log x)$ have on the interval $0 < x < 1$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 10 **(E)** infinitely many

Intermediate.

6. (2021 AMC 12B, Problem #13) How many values of θ in the interval $0 < \theta \leq 2\pi$ satisfy the following equation?

$$1 - 3\sin\theta + 5\cos 3\theta = 0$$

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

7. (2022 AMC 12A, Problem #17) Suppose a is a real number such that the equation

$$a \cdot (\sin x + \sin(2x)) = \sin(3x)$$

has more than one solution in the interval $(0, \pi)$. The set of all such a can be written in the form $(p, q) \cup (q, r)$, where p, q, and r are real numbers with $p < q < r$. What is $p + q + r$?

(A) -4 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 4

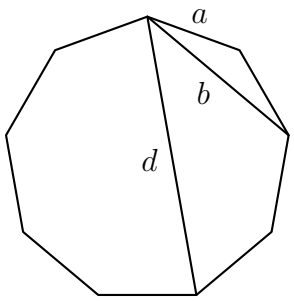
8. (1982 AHSME, Problem #23) The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. The cosine of the smallest angle is

(A) $\frac{3}{4}$ **(B)** $\frac{7}{10}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{9}{14}$ **(E)** none of these

9. (1998 AHSME, Problem #28) In triangle ABC, angle C is a right angle and $CB > CA$. Point D is located on $\overline{BC}$ so that angle CAD is twice angle DAB. If $\frac{AC}{AD} = \frac{2}{3}$, then $\frac{CD}{BD} = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

(A) 10 **(B)** 14 **(C)** 18 **(D)** 22 **(E)** 26

10. (1977 AHSME, Problem #30) If a, b and d are the lengths of a side, a shortest diagonal and a longest diagonal, respectively, of a regular nonagon (see adjoining figure), then



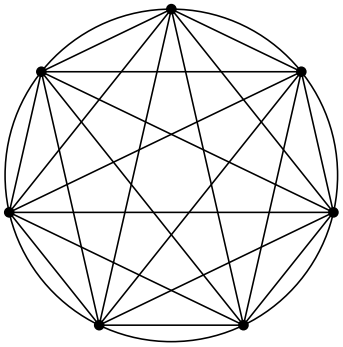
(A) $d = a + b$ **(B)** $d^2 = a^2 + b^2$ **(C)** $d^2 = a^2 + ab + b^2$ **(D)** $b = \frac{a+d}{2}$ **(E)** $b^2 = ad$

Advanced.

11. (2005 AMC 12B, Problem #24) All three vertices of an equilateral triangle are on the parabola $y = x^2$, and one of its sides has a slope of 2. The x-coordinates of the three vertices have a sum of $\dfrac{m}{n}$, where m and n are relatively prime positive integers. What is the value of $m + n$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

12. (2022 AMC 12B, Problem #24) The figure below depicts a regular 7-gon inscribed in a unit circle.



What is the sum of the 4th powers of the lengths of all 21 of its edges and diagonals?
(A) 49 **(B)** 98 **(C)** 147 **(D)** 168 **(E)** 196

13. (1989 AIME, Problem #10) Let a, b, c be the three sides of a triangle, and let α, β, γ be the angles opposite them. If $a^2 + b^2 = 1989c^2$, find

$$\frac{\cot\gamma}{\cot\alpha + \cot\beta}.$$

14. (2002 AIME II, Problem #10) While finding the sine of a certain angle, an absent-minded professor failed to notice that his calculator was not in the correct angular mode. He was lucky to get the right answer. The two least positive real values of x for which the sine of x degrees is the same as the sine of x radians are $\frac{m\pi}{n-\pi}$ and $\frac{p\pi}{q+\pi}$, where m, n, p, and q are positive integers. Find $m+n+p+q$.

15. (2008 AIME II, Problem #8) Let $a = \frac{\pi}{2008}$. Find the smallest positive integer n such that

$$2[\cos(a)\sin(a) + \cos(4a)\sin(2a) + \cos(9a)\sin(3a) + \cdots + \cos(n^2a)\sin(na)]$$

is an integer.

2. Answer Key

Problem	Letter	Value
1	C	$\frac{3}{2}$
2	B	$\frac{3}{5}$
3	B	73
4	D	$\frac{7}{9}$
5	E	infinitely many
6	D	6
7	A	-4
8	A	$\frac{3}{4}$
9	B	14
10	A	$d = a + b$
11	A	14
12	C	147
13		994
14		900
15		251