THE LAWS OF SINES AND COSINES FOR AMC 12: PRACTICE SET

ANDY JO

Contents

1. Problems 2
2. Answer Key 5

1. Problems

Beginner.

1. (2007 AMC 12A, Problem #10) A triangle with side lengths in the ratio $3 : 4 : 5$ is inscribed in a circle of radius 3. What is the area of the triangle?

(A) 8.64 **(B)** 12 **(C)** 5π **(D)** 17.28 **(E)** 18

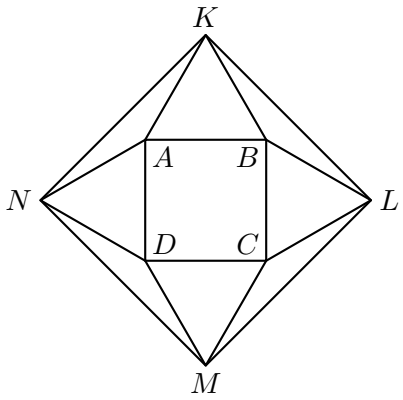
2. (1953 AHSME, Problem #28) In triangle ABC, sides a, b and c are opposite angles A, B and C respectively. $\overline{AD}$ bisects angle A and meets $\overline{BC}$ at D. Then if $x = CD$ and $y = BD$ the correct proportion is:

(A) $\frac{x}{a} = \frac{a}{b+c}$ **(B)** $\frac{x}{b} = \frac{a}{a+c}$ **(C)** $\frac{y}{c} = \frac{c}{b+c}$ **(D)** $\frac{y}{c} = \frac{a}{b+c}$ **(E)** $\frac{x}{y} = \frac{c}{b}$

3. (1958 AHSME, Problem #36) The sides of a triangle are 30, 70, and 80 units. If an altitude is dropped upon the side of length 80, the larger segment cut off on this side is:

(A) 62 **(B)** 63 **(C)** 64 **(D)** 65 **(E)** 66

4. (2003 AMC 12A, Problem #14) Points K, L, M, and N lie in the plane of the square $ABCD$ so that AKB, BLC, CMD, and DNA are equilateral triangles. If $ABCD$ has an area of 16, find the area of $KLMN$.



(A) 32 **(B)** $16 + 16\sqrt{3}$ **(C)** 48 **(D)** $32 + 16\sqrt{3}$ **(E)** 64

5. (2020 AMC 12A, Problem #14) Regular octagon $ABCDEFGH$ has area n. Let m be the area of quadrilateral $ACEG$. What is $\frac{m}{n}$?

(A) $\frac{\sqrt{2}}{4}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{3\sqrt{2}}{5}$ **(E)** $\frac{2\sqrt{2}}{3}$

Intermediate.

6. (1988 AIME, Problem #7) In triangle ABC, $\tan \angle CAB = \frac{22}{7}$, and the altitude from A divides BC into segments of length 3 and 17. What is the area of triangle ABC?

7. (1967 AHSME, Problem #13) A triangle ABC is to be constructed given side a (opposite angle A), angle B, and h_c, the altitude from C. If N is the number of noncongruent solutions, then N

(A) is 1 **(B)** is 2 **(C)** must be zero **(D)** must be infinite
(E) must be zero or infinite

8. (1973 AHSME, Problem #15) A sector with acute central angle θ is cut from a circle of radius 6. The radius of the circle circumscribed about the sector is

(A) $3\cos\theta$ **(B)** $3\sec\theta$ **(C)** $3\cos\frac{1}{2}\theta$ **(D)** $3\sec\frac{1}{2}\theta$ **(E)** 3

9. (2018 AMC 10B, Problem #22) Real numbers x and y are chosen independently and uniformly at random from the interval $[0, 1]$. Which of the following numbers is closest to the probability that x, y, and 1 are the side lengths of an obtuse triangle?

(A) 0.21 **(B)** 0.25 **(C)** 0.29 **(D)** 0.50 **(E)** 0.79

10. (1980 AHSME, Problem #23) Line segments drawn from the vertex opposite the hypotenuse of a right triangle to the points trisecting the hypotenuse have lengths $\sin x$ and $\cos x$, where x is a real number such that $0 < x < \frac{\pi}{2}$. The length of the hypotenuse is

(A) $\frac{4}{3}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{3\sqrt{5}}{5}$ **(D)** $\frac{2\sqrt{5}}{3}$
(E) not uniquely determined by the given information

Advanced.

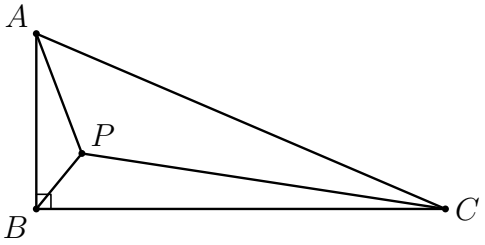
11. (1962 AHSME, Problem #39) The medians AN and BP of a triangle with unequal sides are, respectively, 3 inches and 6 inches long. Its area is $3\sqrt{15}$ square inches. The length of the third median, in inches, is:

(A) 4 **(B)** $3\sqrt{3}$ **(C)** $3\sqrt{6}$ **(D)** $6\sqrt{3}$ **(E)** $6\sqrt{6}$

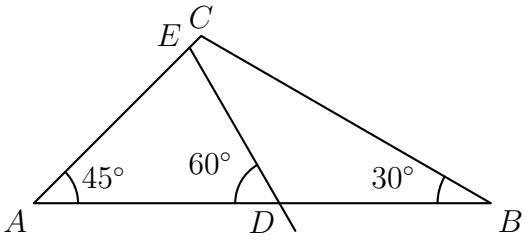
12. (2002 AMC 12A, Problem #23) In triangle ABC, side $\overline{AC}$ and the perpendicular bisector of $\overline{BC}$ meet in point D, and $\overline{BD}$ bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

(A) 14 **(B)** 21 **(C)** 28 **(D)** $14\sqrt{5}$ **(E)** $28\sqrt{5}$

13. (1987 AIME, Problem #9) Triangle ABC has a right angle at B and contains a point P for which $PA = 10$, $PB = 6$, and $\angle APB = \angle BPC = \angle CPA$. Find PC.



14. (1987 AHSME, Problem #30) In the figure, $\triangle ABC$ has $\angle A = 45^\circ$ and $\angle B = 30^\circ$. A line DE, with D on AB and $\angle ADE = 60^\circ$, divides $\triangle ABC$ into two pieces of equal area. (Note: the figure may not be accurate; perhaps E is on CB instead of AC.) The ratio $\frac{AD}{AB}$ is



(A) $\frac{1}{\sqrt{2}}$ **(B)** $\frac{2}{2+\sqrt{2}}$ **(C)** $\frac{1}{\sqrt{3}}$ **(D)** $\frac{1}{\sqrt[3]{6}}$ **(E)** $\frac{1}{\sqrt[4]{12}}$

15. (2023 AIME II, Problem #3) Let $\triangle ABC$ be an isosceles triangle with $\angle A = 90°$. There exists a point P inside $\triangle ABC$ such that $\angle PAB = \angle PBC = \angle PCA$ and $AP = 10$. Find the area of $\triangle ABC$.

2. Answer Key

Problem	Letter	Value
1	A	8.64
2	D	$\frac{y}{c} = \frac{a}{b+c}$
3	D	65
4	D	$32 + 16\sqrt{3}$
5	B	$\frac{\sqrt{2}}{2}$
6		110
7	E	must be zero or infinite
8	D	$3 \sec \frac{1}{2}\theta$
9	C	0.29
10	C	$\frac{3\sqrt{5}}{5}$
11	C	$3\sqrt{6}$
12	D	$14\sqrt{5}$
13		33
14	E	$\frac{1}{\sqrt[4]{12}}$
15		250