TRIGONOMETRIC RATIOS AND THE UNIT CIRCLE FOR AMC 12: PRACTICE SET

ANDY JO

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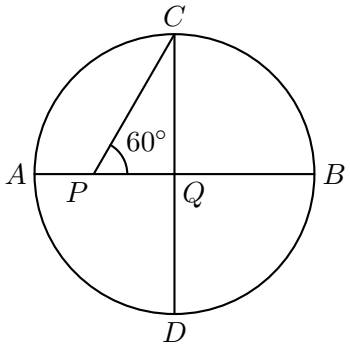
1. Problems

Beginner.

1. (1966 AHSME, Problem #6) AB is a diameter of a circle centered at O. C is a point on the circle such that angle BOC is $60°$. If the diameter of the circle is 5 inches, the length of chord AC, expressed in inches, is:

(A) 3 **(B)** $\frac{5\sqrt{2}}{2}$ **(C)** $\frac{5\sqrt{3}}{2}$ **(D)** $3\sqrt{3}$ **(E)** none of these

2. (1980 AHSME, Problem #5) If AB and CD are perpendicular diameters of circle Q, P in AQ, and $\angle QPC = 60°$, then the length of PQ divided by the length of AQ is



(A) $\frac{\sqrt{3}}{2}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{\sqrt{2}}{2}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

3. (1965 AHSME, Problem #2) A regular hexagon is inscribed in a circle. The ratio of the length of a side of the hexagon to the length of the shorter of the arcs intercepted by the side is:

(A) $1:1$ **(B)** $1:6$ **(C)** $1:\pi$ **(D)** $3:\pi$ **(E)** $6:\pi$

4. (2002 AMC 10B, Problem #24) Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

(A) 5 **(B)** 6 **(C)** 7.5 **(D)** 10 **(E)** 15

5. (1980 AHSME, Problem #18) If $b > 1$, $\sin x > 0$, $\cos x > 0$ and $\log_b \sin x = a$, then $\log_b \cos x$ equals

(A) $2\log_b(1 - b^{\frac{a}{2}})$ **(B)** $\sqrt{1 - a^2}$ **(C)** b^{a^2} **(D)** $\frac{1}{2}\log_b(1 - b^{2a})$
(E) none of these

Intermediate.

6. (1991 AHSME, Problem #21) If

$$f\left(\frac{x}{x-1}\right) = \frac{1}{x}$$

for all $x \notin \{0, 1\}$, and $0 < \theta < \frac{\pi}{2}$, then $f(\sec^2\theta) =$

(A) $\sin^2\theta$ **(B)** $\cos^2\theta$ **(C)** $\tan^2\theta$ **(D)** $\cot^2\theta$ **(E)** $\csc^2\theta$

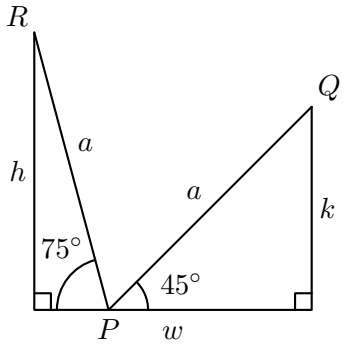
7. (2017 AMC 12A, Problem #15) Let $f(x) = \sin x + 2\cos x + 3\tan x$, using radian measure for the variable x. In what interval does the smallest positive value of x for which $f(x) = 0$ lie?

(A) $(0, 1)$ **(B)** $(1, 2)$ **(C)** $(2, 3)$ **(D)** $(3, 4)$ **(E)** $(4, 5)$

8. (2007 AMC 12B, Problem #19) Rhombus $ABCD$, with side length 6, is rolled to form a cylinder of volume 6 by taping $\overline{AB}$ to $\overline{DC}$. What is $\sin(\angle ABC)$?

(A) $\frac{\pi}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{4}$ **(E)** $\frac{\sqrt{3}}{2}$

9. (1982 AHSME, Problem #22) In a narrow alley of width w a ladder of length a is placed with its foot at a point P between the walls. Resting against one wall at Q, a distance k above the ground, the ladder makes a $45°$ angle with the ground. Resting against the other wall at R, a distance h above the ground, the ladder makes a $75°$ angle with the ground. The width w is equal to



(A) a **(B)** RQ **(C)** k **(D)** $\frac{h+k}{2}$ **(E)** h

10. (1979 AHSME, Problem #24) A polygon is called "simple" if it is not self-intersecting. Sides AB, BC, and CD of simple quadrilateral $ABCD$ have lengths 4, 5, and 20, respectively. If vertex angles B and C are obtuse and $\sin C = -\cos B = \frac{3}{5}$, then side AD has length

(A) 24 **(B)** 24.5 **(C)** 24.6 **(D)** 24.8 **(E)** 25

Advanced.

11. (1977 AHSME, Problem #16) If $i^2 = -1$, then the sum

$$\cos 45° + i\cos 135° + \cdots + i^n \cos(45 + 90n)° + \cdots + i^{40}\cos 3645°$$

equals

(A) $\frac{\sqrt{2}}{2}$ **(B)** $-10i\sqrt{2}$ **(C)** $\frac{21\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{2}(21 - 20i)$ **(E)** $\frac{\sqrt{2}}{2}(21 + 20i)$

12. (2006 AIME II, Problem #1) In convex hexagon $ABCDEF$, all six sides are congruent, $\angle A$ and $\angle D$ are right angles, and $\angle B$, $\angle C$, $\angle E$, and $\angle F$ are congruent. The area of the hexagonal region is $2116(\sqrt{2} + 1)$. Find AB.

13. (2007 AIME I, Problem #4) Three planets revolve about a star in coplanar circular orbits with the star at the center. All planets revolve in the same direction, each at a constant speed, and the periods of their orbits are 60, 84, and 140 years. The positions of the star and all three planets are currently collinear. They will next be collinear after n years. Find n.

14. (1985 AHSME, Problem #10) An arbitrary circle can intersect the graph of $y = \sin x$ in

(A) at most 2 points **(B)** at most 4 points **(C)** at most 6 points
(D) at most 8 points **(E)** more than 16 points

15. (2001 AIME I, Problem #4) In triangle ABC, angles A and B measure 60 degrees and 45 degrees, respectively. The bisector of angle A intersects $\overline{BC}$ at T, and $AT = 24$. The area of triangle ABC can be written in the form $a + b\sqrt{c}$, where a, b, and c are positive integers, and c is not divisible by the square of any prime. Find $a + b + c$.

2. Answer Key

Problem	Letter	Value
1	C	$\frac{5\sqrt{3}}{2}$
2	B	$\frac{\sqrt{3}}{3}$
3	D	$3 : \pi$
4	D	10
5	D	$\frac{1}{2}\log_b(1 - b^{2a})$
6	A	$\sin^2\theta$
7	D	$(3, 4)$
8	A	$\frac{\pi}{9}$
9	E	h
10	E	25
11	D	$\frac{\sqrt{2}}{2}(21 - 20i)$
12		46
13		105
14	E	more than 16 points
15		291