INCLUSION-EXCLUSION AND DERANGEMENTS FOR AMC 12: PRACTICE SET

ANDY JO

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1. Problems

Beginner.

1. (2005 AMC 10B, Problem #13) How many numbers between 1 and 2005 are integer multiples of 3 or 4 but not 12?

(A) 501 **(B)** 668 **(C)** 835 **(D)** 1002 **(E)** 1169

2. (2006 AMC 10A, Problem #21) How many four-digit positive integers have at least one digit that is a 2 or a 3?

(A) 2439 **(B)** 4096 **(C)** 4903 **(D)** 4904 **(E)** 5416

3. Six students take a quiz. The teacher hands the six quizzes back at random, one to each student. In how many ways can the quizzes be handed back so that at least two students receive their own quizzes?

4. The numbers 1, 2, 3, 4, 5, 6 are written in a row in some order. In how many orders is 1 not in the first position, 2 not in the second position, and 3 not in the third position?

5. (2002 AIME I, Problem #1) Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that each three-letter three-digit arrangement is equally likely, the probability that such a license plate will contain at least one palindrome (a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left) is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Intermediate.

6. (2004 AMC 12B, Problem #20) Each face of a cube is painted either red or blue, each with probability $\frac{1}{2}$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

(A) $\frac{1}{4}$ **(B)** $\frac{5}{16}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

7. (2012 AMC 10B, Problem #24) Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

8. (1996 AIME, Problem #6) In a five-team tournament, each team plays one game with every other team. Each team has a 50% chance of winning any game it plays. There are no ties. Let $\frac{m}{n}$ be the probability that the tournament will produce neither an undefeated team nor a winless team, where m and n are relatively prime positive integers. Find $m+n$.

9. Nine friends, one of whom is Alice, each bring a gift to a party. The nine gifts are collected and handed out again so that each friend receives exactly one gift and no friend receives his or her own gift. In how many ways can the gifts be handed out so that the friend whose gift Alice receives does not receive Alice's gift?

10. (2004 AIME II, Problem #4) How many positive integers less than 10,000 have at most two different digits?

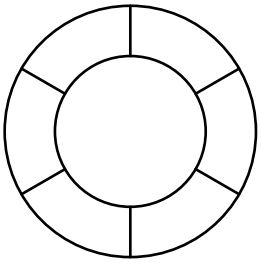
Advanced.

11. (1992 AIME, Problem #5) Let S be the set of all rational numbers r, $0 < r < 1$, that have a repeating decimal expansion in the form

$$0.abcabcabc\ldots = 0.\overline{abc},$$

where the digits a, b, and c are not necessarily distinct. To write the elements of S as fractions in lowest terms, how many different numerators are required?

12. (2016 AIME II, Problem #12) The figure below shows a ring made of six small sections which you are to paint on a wall. You have four paint colors available and will paint each of the six sections a solid color. Find the number of ways you can choose to paint the sections if no two adjacent sections can be painted with the same color.



13. (2016 AMC 10B, Problem #25) Let

$$f(x) = \sum_{k=2}^{10} (\lfloor kx \rfloor - k\lfloor x \rfloor),$$

where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to r. How many distinct values does $f(x)$ assume for $x \geq 0$?

(A) 32 **(B)** 36 **(C)** 45 **(D)** 46 **(E)** infinitely many

14. (2010 AMC 12A, Problem #25) Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

(A) 560 **(B)** 564 **(C)** 568 **(D)** 1498 **(E)** 2255

15. (2015 AIME II, Problem #12) There are $2^{10} = 1024$ possible 10-letter strings in which each letter is either an A or a B. Find the number of such strings that do not have more than 3 adjacent letters that are identical.

2. Answer Key

Problem	Letter	Value
1	C	835
2	E	5416
3		191
4		426
5		59
6	B	$\frac{5}{16}$
7	B	132
8		49
9		118664
10		927
11		660
12		732
13	A	32
14	C	568
15		548