INCLUSION-EXCLUSION AND DERANGEMENTS FOR AMC 12: PRACTICE SET SOLUTIONS

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1. Solutions

Beginner.

1. (2005 AMC 10B, Problem #13) How many numbers between 1 and 2005 are integer multiples of 3 or 4 but not 12?

(A) 501 **(B)** 668 **(C)** 835 **(D)** 1002 **(E)** 1169

Strategy. Count the multiples of 3 or 4 with inclusion-exclusion. The numbers divisible by both 3 and 4 are exactly the multiples of 12, and these are the numbers to remove.

Solution. Neither 1 nor 2005 is a multiple of 3 or 4, so it makes no difference whether the endpoints are included. Among the integers from 1 to 2005 there are

$$\left\lfloor \frac{2005}{3} \right\rfloor = 668 \text{ multiples of } 3 \quad \text{and} \quad \left\lfloor \frac{2005}{4} \right\rfloor = 501 \text{ multiples of } 4.$$

A number is a multiple of both 3 and 4 exactly when it is a multiple of 12, and there are $\lfloor 2005/12 \rfloor = 167$ of these. By inclusion-exclusion, the number of multiples of 3 or 4 is

$$668 + 501 - 167 = 1002.$$

Every multiple of 12 is among these 1002 numbers, so removing them leaves $1002 - 167 = 835$.

Answer: (C) 835. □

2. (2006 AMC 10A, Problem #21) How many four-digit positive integers have at least one digit that is a 2 or a 3?

(A) 2439 **(B)** 4096 **(C)** 4903 **(D)** 4904 **(E)** 5416

Strategy. Count the complement: the four-digit integers that use neither 2 nor 3.

Solution. There are 9000 four-digit positive integers, from 1000 to 9999. Count those with no digit equal to 2 or 3. The first digit cannot be 0, 2, or 3, so it has 7 choices. Each of the other three digits can be any of the 8 digits other than 2 and 3. This gives $7 \cdot 8^3 = 3584$ integers with no 2 and no 3. All the others have at least one digit equal to 2 or 3, so the answer is

$$9000 - 3584 = 5416.$$

Answer: (E) 5416. □

3. Six students take a quiz. The teacher hands the six quizzes back at random, one to each student. In how many ways can the quizzes be handed back so that at least two students receive their own quizzes?

Strategy. Count the complement: the ways in which no student, or exactly one student, receives his or her own quiz. The Fixed Points theorem gives both numbers.

Solution. Number the students and their quizzes 1 to 6, and let a_i be the quiz handed to student i. The ways of handing back the quizzes are the $6! = 720$ permutations $(a_1, \ldots, a_6)$ of $1, \ldots, 6$, and student i receives his or her own quiz exactly when i is a fixed point. By the Fixed Points theorem, exactly k students receive their own quizzes in $\binom{6}{k} D_{6-k}$ ways. Since $D_6 = 265$ and $D_5 = 44$, no student receives his or her own quiz in $\binom{6}{0} D_6 = 265$ ways, and exactly one student does in $\binom{6}{1} D_5 = 6 \cdot 44 = 264$ ways. In every other way, at least two students receive their own quizzes, so the number of ways is

$$720 - 265 - 264 = 191.$$

Answer: 191. □

4. The numbers 1, 2, 3, 4, 5, 6 are written in a row in some order. In how many orders is 1 not in the first position, 2 not in the second position, and 3 not in the third position?

Strategy. For $i = 1, 2, 3$, consider the orders with i in position i. Apply inclusion-exclusion in the symmetric case, as in the proof of the formula for D_n, with only three of the six numbers kept out of their own positions.

Solution. There are $6! = 720$ orders. For $i = 1, 2, 3$, let A_i be the set of orders with i in position i; the required orders are those in none of A_1, A_2, A_3. Fix t of the numbers 1, 2, 3. In an order in which each of these t numbers is in its own position, the other $6 - t$ numbers fill the other $6 - t$ positions in any order, so there are $(6 - t)!$ such orders. Hence every intersection of t of the sets has $(6 - t)!$ elements, and

$$s_1 = \binom{3}{1} \cdot 5! = 360, \qquad s_2 = \binom{3}{2} \cdot 4! = 72, \qquad s_3 = \binom{3}{3} \cdot 3! = 6.$$

By inclusion-exclusion, the number of orders in none of the sets is

$$720 - 360 + 72 - 6 = 426.$$

Answer: 426. □

5. (2002 AIME I, Problem #1) Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that each three-letter three-digit arrangement is equally likely, the probability that such a license plate will contain at least one palindrome (a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left) is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Strategy. Count the plates whose letters read the same in both directions and the plates whose digits do, and combine the two counts by inclusion-exclusion.

Solution. There are $26^3 \cdot 10^3$ plates, all equally likely. A three-letter arrangement reads the same in both directions exactly when its first and third letters agree, so a palindrome is determined by its first two letters, and 26^2 of the arrangements are palindromes. In the same way, 10^2 of the three-digit arrangements are palindromes. Let A be the set of plates whose letters form a palindrome and B the set of plates whose digits form a palindrome. By the Multiplication Principle,

$$|A| = 26^2 \cdot 10^3, \qquad |B| = 26^3 \cdot 10^2, \qquad |A \cap B| = 26^2 \cdot 10^2,$$

so by inclusion-exclusion

$$|A \cup B| = 26^2 \cdot 10^2 \cdot (10 + 26 - 1) = 26^2 \cdot 10^2 \cdot 35.$$

The probability is

$$\frac{26^2 \cdot 10^2 \cdot 35}{26^3 \cdot 10^3} = \frac{35}{260} = \frac{7}{52},$$

which is in lowest terms, so $m + n = 7 + 52 = 59$.

Answer: 59. □

Intermediate.

6. (2004 AMC 12B, Problem #20) Each face of a cube is painted either red or blue, each with probability $\frac{1}{2}$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

(A) $\frac{1}{4}$ **(B)** $\frac{5}{16}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

Strategy. The cube can be placed as required exactly when, for some pair of opposite faces, the four faces outside that pair have one color. Use one set for each of the three pairs of opposite faces and apply inclusion-exclusion for three sets.

Solution. The faces are colored independently, each red or blue with probability $\frac{1}{2}$, so the $2^6 = 64$ colorings are equally likely. When the cube rests on a face, its top and bottom faces are opposite, and the vertical faces are the other four. So the cube can be placed as required exactly when, for one of the three pairs of opposite faces, the four faces outside that pair have the same color. For $i = 1, 2, 3$, let A_i be the set of colorings in which the four faces outside the ith pair have one color.

The four faces outside the pair have one of 2 colors, and each of the two faces of the pair is either color, so $|A_i| = 2 \cdot 2^2 = 8$. For $i \neq j$, the four faces outside the ith pair and the four faces outside the jth pair share the two faces of the third pair, and together they include all six faces. So in a coloring in $A_i \cap A_j$ all six faces have the same color, and $|A_i \cap A_j| = 2$; in the same way, $|A_1 \cap A_2 \cap A_3| = 2$. By inclusion-exclusion,

$$|A_1 \cup A_2 \cup A_3| = 3 \cdot 8 - 3 \cdot 2 + 2 = 20,$$

and the probability is $\frac{20}{64} = \frac{5}{16}$.

Answer: (B) $\frac{5}{16}$. □

7. (2012 AMC 10B, Problem #24) Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

Strategy. Sort each song by the set of girls who like it. The second condition says that three particular types of song must each occur at least once, and inclusion-exclusion counts that.

Solution. For each song, the set of girls who like it is one of seven types: nobody, Amy only, Beth only, Jo only, Amy and Beth, Amy and Jo, or Beth and Jo. The eighth possibility, all three girls, is ruled out by the first condition. A way for the girls to like the songs is a choice of type for each of the four different songs, so there are $7^4 = 2401$ ways that satisfy the first condition.

The second condition says that each of the types "Amy and Beth," "Amy and Jo," and "Beth and Jo" is used by at least one song. Let X_1, X_2, and X_3 be the sets of choices in which the first, second, or third of these types is not used. Leaving out one type leaves 6 types for each song, so $|X_i| = 6^4 = 1296$. In the same way $|X_i \cap X_j| = 5^4 = 625$ for $i \neq j$, and $|X_1 \cap X_2 \cap X_3| = 4^4 = 256$. By inclusion-exclusion,

$$|X_1 \cup X_2 \cup X_3| = 3 \cdot 1296 - 3 \cdot 625 + 256 = 3888 - 1875 + 256 = 2269.$$

The number of ways that use all three types is $2401 - 2269 = 132$.

Answer: (B) 132. □

8. (1996 AIME, Problem #6) In a five-team tournament, each team plays one game with every other team. Each team has a 50% chance of winning any game it plays. There are no ties. Let $\frac{m}{n}$ be the probability that the tournament will produce neither an undefeated team nor a winless team, where m and n are relatively prime positive integers. Find $m+n$.

Strategy. Apply inclusion-exclusion to the ten sets of outcomes in which a given team is undefeated or a given team is winless. Sort the intersections into types; most of them are empty, because exactly one of two teams wins the game between them.

Solution. The $\binom{5}{2} = 10$ games have $2^{10} = 1024$ equally likely outcomes. For each team T, let U_T be the set of outcomes in which T wins all four of its games and W_T the set of outcomes in which T loses all four. Each of these ten sets fixes the results of the four games of one team and leaves the other six games free, so it has $2^6 = 64$ elements, and $s_1 = 10 \cdot 64 = 640$.

Sort the intersections of two of the sets into types.

- Two different teams cannot both be undefeated, or both be winless, since one of them wins the game between them and the other loses it. A team cannot be both undefeated and winless. So these intersections are empty.
- For different teams T and T', an outcome in $U_T \cap W_{T'}$ has T winning its four games and T' losing its four games. These conditions agree on the game between T and T', so together they fix $4 + 4 - 1 = 7$ games, and the intersection has $2^3 = 8$ elements. There are $5 \cdot 4 = 20$ such intersections, so $s_2 = 20 \cdot 8 = 160$.

Any three of the ten sets include two sets U_T and $U_{T''}$, or two sets W_T and $W_{T''}$, with $T \neq T''$, so every intersection of three or more of the sets is empty. By inclusion-exclusion, $640 - 160 = 480$ outcomes have an undefeated team or a winless team, so $1024 - 480 = 544$ have neither. The probability is $\frac{544}{1024} = \frac{17}{32}$, and $m + n = 17 + 32 = 49$.

Answer: 49. □

9. Nine friends, one of whom is Alice, each bring a gift to a party. The nine gifts are collected and handed out again so that each friend receives exactly one gift and no friend receives his or her own gift. In how many ways can the gifts be handed out so that the friend whose gift Alice receives does not receive Alice's gift?

Strategy. Number Alice 1, and let c be the friend whose gift she receives. The condition says that c does not receive gift 1, which is the second case in the proof of the Derangement Recursion: renaming gift 1 as gift c gives a derangement of eight numbers.

Solution. Number the friends 1 to 9, with Alice as friend 1, and give each gift its owner's number. Let a_i be the gift that friend i receives. The ways of handing out the gifts are the derangements $(a_1, \ldots, a_9)$ of $1, \ldots, 9$. Alice receives the gift of friend $c = a_1$, where $c \neq 1$, and the condition is $a_c \neq 1$.

Fix one of the 8 possible values of c. Since $a_1 = c$, the entries $a_2, \ldots, a_9$ are the eight numbers other than c. Rename the number 1 as c in this list. Then the conditions $a_c \neq 1$ and $a_i \neq i$ for $i \neq 1, c$ say exactly that no entry of the renamed list $a_2, \ldots, a_9$ equals its position, so the renamed list is a derangement of $2, 3, \ldots, 9$. Renaming c back as 1 reverses this step, so the derangements with $a_1 = c$ and $a_c \neq 1$ correspond one to one to the derangements of eight numbers, and there are $D_8 = 14833$ of them. Hence the number of ways is

$$8 \cdot D_8 = 8 \cdot 14833 = 118664.$$

Answer: 118664. □

10. (2004 AIME II, Problem #4) How many positive integers less than 10,000 have at most two different digits?

Strategy. Sort the numbers by their number of digits and by the digits they use. Writing a number of k digits with exactly two given digits places its k positions into two boxes, one for each digit, with neither box empty.

Solution. The numbers have 1 to 4 digits. A number with only one distinct digit repeats one of the digits 1 to 9, so there are 9 such numbers of each length and 36 in all.

Now count the numbers with k digits, $2 \le k \le 4$, and exactly two different digits. Once the two digits are chosen, choosing the digit in each position places the k positions into two boxes, one for each digit, and both digits appear exactly when neither box is empty.

- If both digits are nonzero, they can be chosen in $\binom{9}{2} = 36$ ways, and each choice gives $2^k - 2$ numbers, the number of distributions of k objects into two boxes with no box empty.
- If one digit is 0, the other digit d can be chosen in 9 ways. The first position must hold d, and the other $k-1$ positions hold 0 or d with 0 used at least once, which gives $2^{k-1} - 1$ numbers.

So there are $36 \cdot (2^k - 2) + 9 \cdot (2^{k-1} - 1)$ numbers with k digits: $36 \cdot 2 + 9 \cdot 1 = 81$ for $k = 2$, $36 \cdot 6 + 9 \cdot 3 = 243$ for $k = 3$ and $36 \cdot 14 + 9 \cdot 7 = 567$ for $k = 4$. The total is

$$36 + 81 + 243 + 567 = 927.$$

Answer: 927. □

Advanced.

11. (1992 AIME, Problem #5) Let S be the set of all rational numbers r, $0 < r < 1$, that have a repeating decimal expansion in the form

$$0.abcabcabc\ldots = 0.\overline{abc},$$

where the digits a, b, and c are not necessarily distinct. To write the elements of S as fractions in lowest terms, how many different numerators are required?

Strategy. The elements of S are the fractions $\frac{k}{999}$ with $1 \le k \le 998$. A positive integer m is a required numerator exactly when $\frac{m}{n}$ is in lowest terms for some divisor $n > m$ of 999. Count the m relatively prime to 999 with Euler's function, then find the other numerators.

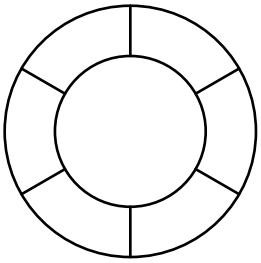
Solution. Let $x = 0.\overline{abc}$ and $k = 100a + 10b + c$. Then $1000x = k + x$, so $x = \frac{k}{999}$, and the condition $0 < x < 1$ means $1 \le k \le 998$. So the elements of S are the fractions $\frac{k}{999}$ with $1 \le k \le 998$. In lowest terms, $\frac{k}{999} = \frac{m}{n}$, where n divides 999, $1 \le m < n$ and $\gcd(m, n) = 1$. Conversely, if n divides 999, $1 \le m < n$ and $\gcd(m, n) = 1$, then $\frac{m}{n} = \frac{k}{999}$ with $k = \frac{999m}{n}$ and $1 \le k \le 998$, so $\frac{m}{n}$ is an element of S in lowest terms. So m is a required numerator exactly when $m < n$ and $\gcd(m, n) = 1$ for some divisor n of $999 = 3^3 \cdot 37$.

- If m is relatively prime to 999, take $n = 999$. By the formula for Euler's function, there are
$$\varphi(999) = 999 \cdot \frac{2}{3} \cdot \frac{36}{37} = 648$$
such numerators, all less than 999.
- If 3 divides m, then n is a divisor of 999 greater than 1 and not divisible by 3, so $n = 37$. Then $m < 37$, and every multiple of 3 less than 37 is relatively prime to 37. This gives the 12 numerators $3, 6, \ldots, 36$.
- If 37 divides m, then n is one of 1, 3, 9, 27, all less than m, so there is no such numerator.

The number of different numerators is $648 + 12 = 660$.

Answer: 660. □

12. (2016 AIME II, Problem #12) The figure below shows a ring made of six small sections which you are to paint on a wall. You have four paint colors available and will paint each of the six sections a solid color. Find the number of ways you can choose to paint the sections if no two adjacent sections can be painted with the same color.



Strategy. Apply inclusion-exclusion to the six pairs of adjacent sections. Requiring some pairs to match joins the sections into blocks of one color, so the size of an intersection depends only on the number of pairs: $j \le 5$ pairs leave $6-j$ blocks, and all six pairs leave one block.

Solution. Number the sections 1 to 6 around the ring. The pairs of adjacent sections are $\{1,2\}$, $\{2,3\}$, $\{3,4\}$, $\{4,5\}$, $\{5,6\}$ and $\{6,1\}$; for each of them, consider the set of colorings in which its two sections have the same color. There are 4^6 colorings in all, and the required colorings are those in none of the six sets.

Choose j of the pairs, with $j \le 5$. Going around the ring, the $6-j \ge 1$ pairs not chosen are $6-j$ of the six boundaries between neighboring sections, and they cut the ring into $6-j$ blocks of consecutive sections; within a block, neighboring sections form chosen pairs. A coloring lies in all j chosen sets exactly when each block has one color, so the intersection has 4^{6-j} colorings. If all six pairs are chosen, all six sections have one color, so the intersection has 4 colorings, not $4^0 = 1$. For each j, all the intersections of j of the sets have the same size, so by inclusion-exclusion in the symmetric case the number of colorings is

$$\sum_{j=0}^{5}(-1)^j\binom{6}{j}4^{6-j} + 4 = 4096 - 6144 + 3840 - 1280 + 240 - 24 + 4 = 732.$$

The binomial theorem gives the same value: the sum over $j = 0, 1, \ldots, 6$ is $(4-1)^6 = 729$, its term for $j = 6$ is 1, and $729 - 1 + 4 = 732$.

Answer: 732. □

13. (2016 AMC 10B, Problem #25) Let

$$f(x) = \sum_{k=2}^{10}(\lfloor kx \rfloor - k\lfloor x \rfloor),$$

where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to r. How many distinct values does $f(x)$ assume for $x \ge 0$?

(A) 32 **(B)** 36 **(C)** 45 **(D)** 46 **(E)** infinitely many

Strategy. Write $x = n + t$ with $n = \lfloor x \rfloor$ and $0 \le t < 1$. Then $f(x)$ depends only on t, and it increases exactly when t reaches a fraction with denominator from 2 to 10; count these fractions with Euler's function.

Solution. Let $n = \lfloor x \rfloor$ and $t = x - n$, so $0 \le t < 1$. Since kn is an integer, $\lfloor kx \rfloor = \lfloor kn + kt \rfloor = kn + \lfloor kt \rfloor$, so each term of the sum is $\lfloor kt \rfloor$ and

$$f(x) = \sum_{k=2}^{10} \lfloor kt \rfloor.$$

Every t with $0 \le t < 1$ occurs (take $x = t$), so it suffices to count the values of this sum for $0 \le t < 1$. As t increases, each $\lfloor kt \rfloor$ never decreases, and it increases exactly at the numbers $t = \frac{j}{k}$ with $1 \le j \le k-1$. So the sum is constant between consecutive numbers of the form $\frac{j}{k}$ with $2 \le k \le 10$ and $1 \le j < k$, and it increases at each of them. If there are N such numbers, they cut the interval $0 \le t < 1$ into $N+1$ pieces, on which the sum takes $N+1$ different values.

In lowest terms, each of these numbers is $\frac{a}{b}$ with $2 \le b \le 10$, $1 \le a < b$ and $\gcd(a,b) = 1$, and each such fraction is one of them. For a given b, the numerators a are the integers among $1, \ldots, b$ relatively prime to b, since b itself is not, so there are $\varphi(b)$ of them. Hence

$$N = \varphi(2) + \varphi(3) + \cdots + \varphi(10) = 1 + 2 + 2 + 4 + 2 + 6 + 4 + 6 + 4 = 31,$$

and $f(x)$ takes 32 distinct values.

Answer: (A) 32. □

14. (2010 AMC 12A, Problem #25) Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

(A) 560 **(B)** 564 **(C)** 568 **(D)** 1498 **(E)** 2255

Strategy. Describe a quadrilateral by the list of its sides in counterclockwise order. Count the lists with stars and bars and an upper bound, then count them up to the choice of the first side by equal overcounting.

Solution. Two facts about quadrilaterals are used. Each side of a quadrilateral is shorter than the path along the other three sides, so it is less than their sum. Conversely, if four positive lengths are given in order and each is less than the sum of the other three, there is a convex cyclic quadrilateral whose sides, read counterclockwise, are these lengths in this order, and any two such quadrilaterals are related by a rotation and a translation.

Read the sides of a quadrilateral counterclockwise as a list (a, b, c, d), starting from any side. A rotation and a translation keep the counterclockwise order, and a different starting side shifts the list cyclically. So by the two facts, the quadrilaterals to be counted correspond to the lists of positive integers with $a + b + c + d = 32$ in which each entry is less than the sum of the other three, two lists being the same when one is a cyclic shift of the other. The other three entries add up to 32 minus the entry, so the condition says that every entry is at most 15.

To count the lists, subtract 1 from each entry: this gives nonnegative integers with sum 28, each at most 14. Two of them cannot both be at least 15, since $30 > 28$, so by the Upper Bounds theorem with $k = 4$, $n = 28$ and $m = 14$ there are

$$\binom{31}{3} - \binom{4}{1}\binom{16}{3} = 4495 - 4 \cdot 560 = 2255$$

lists. A cyclic shift of a list of the form (a, b, a, b) has the same form. There are 15 such lists, with $a + b = 16$: the list $(8, 8, 8, 8)$ is a class by itself, and the other 14 form 7 classes $\{(a, b, a, b), (b, a, b, a)\}$ with $a \ne b$. If two cyclic shifts of a list are equal, the list equals its own shift by one, two or three places, and then it has the form (a, b, a, b). So each of the

other $2255 - 15 = 2240$ lists has four different cyclic shifts, all among these lists, and by equal overcounting they form $\frac{2240}{4} = 560$ classes. The number of quadrilaterals is

$$560 + 7 + 1 = 568.$$

Answer: (C) 568. □

15. (2015 AIME II, Problem #12) There are $2^{10} = 1024$ possible 10-letter strings in which each letter is either an A or a B. Find the number of such strings that do not have more than 3 adjacent letters that are identical.

Strategy. A string is determined by its first letter and its run lengths. The condition says that every run has length at most 3, so count the lists of run lengths for each number of runs with the Upper Bounds theorem.

Solution. The condition says that no 4 adjacent letters are identical, that is, every run has length 1, 2 or 3. A string of A's and B's is determined by its first letter and its list of run lengths, since the runs alternate letters. So the answer is 2 times the number of lists $(x_1, \ldots, x_r)$ of integers with $1 \le x_i \le 3$ and $x_1 + \cdots + x_r = 10$, over all r. Since $10 = x_1 + \cdots + x_r \le 3r$, there are at least 4 runs. Put $y_i = x_i - 1$; then $0 \le y_i \le 2$ and $y_1 + \cdots + y_r = 10 - r$. By the Upper Bounds theorem with $k = r$, $n = 10 - r$ and $m = 2$, the number of lists with r runs is

$$\sum_j (-1)^j \binom{r}{j} \binom{9 - 3j}{r - 1},$$

where the sum runs over the integers j with $0 \le j \le r$ and $3j \le 10 - r$. For $r = 4, 5, \ldots, 10$ this gives the following numbers.

Runs r	Lists of run lengths
4	$\binom{9}{3} - 4\binom{6}{3} + 6\binom{3}{3} = 84 - 80 + 6 = 10$
5	$\binom{9}{4} - 5\binom{6}{4} = 126 - 75 = 51$
6	$\binom{9}{5} - 6\binom{6}{5} = 126 - 36 = 90$
7	$\binom{9}{6} - 7\binom{6}{6} = 84 - 7 = 77$
8	$\binom{9}{7} = 36$
9	$\binom{9}{8} = 9$
10	$\binom{9}{9} = 1$

There are $10 + 51 + 90 + 77 + 36 + 9 + 1 = 274$ lists, so the number of strings is $2 \cdot 274 = 548$.

Answer: 548. □

2. Answer Key

Problem	Letter	Value
1	C	835
2	E	5416
3		191
4		426
5		59
6	B	$\frac{5}{16}$
7	B	132
8		49
9		118664
10		927
11		660
12		732
13	A	32
14	C	568
15		548