ADVANCED COUNTING AND BIJECTIONS FOR AMC 12: PRACTICE SET

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1. Problems

Beginner.

1. (2018 AMC 10B, Problem #9) The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let p be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability p?

(A) 13 **(B)** 26 **(C)** 32 **(D)** 39 **(E)** 42

2. (2003 AMC 10A, Problem #21) Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

(A) 22 **(B)** 25 **(C)** 27 **(D)** 28 **(E)** 729

3. (1987 AIME, Problem #1) An ordered pair (m, n) of nonnegative integers is called "simple" if adding $m + n$ in base 10 requires no carrying. Find the number of simple ordered pairs that sum to 1492.

4. (2004 AMC 12A, Problem #13) Let S be the set of points (a, b) in the coordinate plane, where each of a and b may be -1, 0, or 1. How many distinct lines pass through at least two members of S?

(A) 8 **(B)** 20 **(C)** 24 **(D)** 27 **(E)** 36

5. (2021 AMC 10A, Problem #15) Values for A, B, C, and D are to be selected from $\{1, 2, 3, 4, 5, 6\}$ without replacement (i.e., no two letters have the same value). How many ways are there to make such choices so that the two curves $y = Ax^2 + B$ and $y = Cx^2 + D$ intersect?
(The order in which the curves are listed does not matter; for example, the choices $A = 3$, $B = 2$, $C = 4$, $D = 1$ is considered the same as the choices $A = 4$, $B = 1$, $C = 3$, $D = 2$.)

(A) 30 **(B)** 60 **(C)** 90 **(D)** 180 **(E)** 360

Intermediate.

6. (2001 AIME I, Problem #6) A fair die is rolled four times. The probability that each of the final three rolls is at least as large as the roll preceding it may be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

7. (2019 AMC 10B, Problem #25) How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

(A) 55 **(B)** 60 **(C)** 65 **(D)** 70 **(E)** 75

8. (1986 AIME, Problem #7) The increasing sequence 1, 3, 4, 9, 10, 12, 13, ... consists of all those positive integers which are powers of 3 or sums of distinct powers of 3. Find the 100th term of this sequence.

9. (2021 Fall AMC 12A, Problem #18) Each of 20 balls is tossed independently and at random into one of 5 bins. Let p be the probability that some bin ends up with 3 balls, another with 5 balls, and the other three with 4 balls each. Let q be the probability that every bin ends up with 4 balls. What is $\frac{p}{q}$?

(A) 1 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 16

10. (2010 AMC 12A, Problem #16) Bernardo randomly picks 3 distinct numbers from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8\}$$

and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

Advanced.

11. (2003 AMC 12A, Problem #22) Objects A and B move simultaneously in the coordinate plane via a sequence of steps, each of length one. Object A starts at $(0, 0)$ and each of its steps is either right or up, both equally likely. Object B starts at $(5, 7)$ and each of its steps is either left or down, both equally likely. Which of the following is closest to the probability that the objects meet?

(A) 0.10 **(B)** 0.15 **(C)** 0.20 **(D)** 0.25 **(E)** 0.30

12. (2008 AMC 12B, Problem #22) A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers choose their spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

(A) $\frac{11}{20}$ **(B)** $\frac{4}{7}$ **(C)** $\frac{81}{140}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{17}{28}$

13. (1986 AIME, Problem #13) In a sequence of coin tosses, one can keep a record of instances in which a tail is immediately followed by a head, a head is immediately followed by a head, and so on. We denote these by TH, HH, and so on. For example, in the sequence TTTHHTHTTTHHTTH of 15 coin tosses, there are two HH, three HT, four TH, and five TT subsequences. How many different sequences of 15 coin tosses contain exactly two HH, three HT, four TH, and five TT subsequences?

14. (2010 AIME II, Problem #8) Let N be the number of ordered pairs of nonempty sets $\mathcal{A}$ and $\mathcal{B}$ that have the following properties:

- $\mathcal{A} \cup \mathcal{B} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$,
- $\mathcal{A} \cap \mathcal{B} = \emptyset$,
- the number of elements of $\mathcal{A}$ is not an element of $\mathcal{A}$,
- the number of elements of $\mathcal{B}$ is not an element of $\mathcal{B}$.

Find N.

15. A one-way street has 6 parking spaces in a row, numbered 1 through 6 in the direction of traffic. Four cars arrive one at a time, and each driver has a preferred space, one of the six. A driver goes to the preferred space and parks in the first empty space at or after it; a driver who finds no such space leaves without parking. Different drivers may prefer the same space. For how many of the 6^4 lists of preferences, one entry for each car in order of arrival, do all four cars park?

2. Answer Key

Problem	Letter	Value
1	D	39
2	D	28
3		300
4	B	20
5	C	90
6		79
7	C	65
8		981
9	E	16
10	B	$\frac{37}{56}$
11	C	0.20
12	E	$\frac{17}{28}$
13		560
14		772
15		1029