ADVANCED COUNTING AND BIJECTIONS FOR AMC 12: PRACTICE SET SOLUTIONS

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Contents

1. Solutions 2
2. Answer Key 10

1. Solutions

Beginner.

1. (2018 AMC 10B, Problem #9) The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let p be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability p?

(A) 13 **(B)** 26 **(C)** 32 **(D)** 39 **(E)** 42

Strategy. Replace the number d on top of every die by $7 - d$. This rule is its own inverse and turns a sum s into $49 - s$.

Solution. Write an outcome as $(d_1, d_2, \ldots, d_7)$, where d_i is the number on top of the ith die. All 6^7 outcomes are equally likely. Replace every d_i by $7 - d_i$. The new entries are again integers from 1 to 6, and doing the replacement twice gives back the original outcome. So the rule is its own inverse, and it is a bijection from the set of outcomes to itself. It changes a sum $s = d_1 + d_2 + \cdots + d_7$ into

$$(7 - d_1) + (7 - d_2) + \cdots + (7 - d_7) = 49 - s.$$

Hence the outcomes with sum 10 are matched one to one with the outcomes with sum $49 - 10 = 39$. The two sums occur in equally many of the equally likely outcomes, so the sum 39 also occurs with probability p.

Answer: (D) 39. □

2. (2003 AMC 10A, Problem #21) Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

(A) 22 **(B)** 25 **(C)** 27 **(D)** 28 **(E)** 729

Strategy. An assortment is a selection with repetition from three kinds of cookies, so stars and bars counts it.

Solution. Let c, o, and p be the numbers of chocolate chip, oatmeal, and peanut butter cookies in the assortment. An assortment is a solution of

$$c + o + p = 6$$

in nonnegative integers, and the tray has enough cookies of each kind for every solution. By stars and bars, with 6 stars and 2 bars, the number of solutions is

$$\binom{6+3-1}{6} = \binom{8}{2} = 28.$$

Answer: (D) 28. □

3. (1987 AIME, Problem #1) An ordered pair (m, n) of nonnegative integers is called "simple" if adding $m + n$ in base 10 requires no carrying. Find the number of simple ordered pairs that sum to 1492.

Strategy. Without carrying, the two digits in each place add up to the digit of 1492 in that place. Choose the split of each digit separately and multiply.

Solution. Since $m+n = 1492$, both numbers are at most 1492. Write each with four digits, using leading zeros. If the addition needs no carrying, then in each place the two digits add up to at most 9, and that total is the digit of the sum in the same place. So in a simple pair with sum 1492, the digits of m and n add up to 1 in the thousands place, 4 in

the hundreds place, 9 in the tens place and 2 in the units place. Conversely, digits with these totals add up to 1492 with no carrying, since every total is at most 9.

A place whose total is d allows $d+1$ choices: the digit of m is one of $0, 1, \ldots, d$, and the digit of n is d minus it. The number of choices in one place does not depend on the choices made in the other places. By the Multiplication Principle, the number of simple pairs is

$$(1+1)(4+1)(9+1)(2+1) = 2 \cdot 5 \cdot 10 \cdot 3 = 300.$$

Answer: 300. □

4. (2004 AMC 12A, Problem #13) Let S be the set of points (a, b) in the coordinate plane, where each of a and b may be -1, 0, or 1. How many distinct lines pass through at least two members of S?

(A) 8 **(B)** 20 **(C)** 24 **(D)** 27 **(E)** 36

Strategy. Describe each line by a pair of points of S on it. A line through three points of S has three such descriptions and any other line has one, so count the two kinds of lines separately.

Solution. Two points of S lie on exactly one line, and each line in question contains at least one pair of points of S. There are $\binom{9}{2} = 36$ pairs. No line contains more than three points of S: a vertical line contains at most the three points of one column, and any other line meets each of the three columns $a = -1$, $a = 0$, $a = 1$ at most once.

Besides the three columns, a line through three points of S passes through one point of each column, say $(-1, u)$, $(0, v)$ and $(1, w)$. Since 0 is halfway between -1 and 1, the middle point is the midpoint of the other two, so $u + w = 2v$. With u, v, w in $\{-1, 0, 1\}$, either $u = v = w$, which gives the three rows, or $v = 0$ and $w = -u \neq 0$, which gives the two diagonals. So exactly 8 lines contain three points of S.

Each of these 8 lines contains $\binom{3}{2} = 3$ pairs, so they account for $8 \cdot 3 = 24$ pairs. Every other line contains exactly two points of S, so the remaining $36 - 24 = 12$ pairs lie on 12 different lines. The number of lines is $8 + 12 = 20$.

Answer: (B) 20. □

5. (2021 AMC 10A, Problem #15) Values for A, B, C, and D are to be selected from $\{1, 2, 3, 4, 5, 6\}$ without replacement (i.e., no two letters have the same value). How many ways are there to make such choices so that the two curves $y = Ax^2 + B$ and $y = Cx^2 + D$ intersect?

(The order in which the curves are listed does not matter; for example, the choices $A = 3$, $B = 2$, $C = 4$, $D = 1$ is considered the same as the choices $A = 4$, $B = 1$, $C = 3$, $D = 2$.)

(A) 30 **(B)** 60 **(C)** 90 **(D)** 180 **(E)** 360

Strategy. First decide when the two curves meet. Then count the ordered choices of (A, B, C, D) with a symmetry that pairs each choice that works with one that does not, and finally divide by 2 because the order of the two curves does not matter.

Solution. The curves meet where $Ax^2 + B = Cx^2 + D$, that is, where $(A - C)x^2 = D - B$. Since $A \neq C$, this says $x^2 = \frac{D-B}{A-C}$, which has a real solution exactly when $\frac{D-B}{A-C} \geq 0$. Since $D \neq B$, the fraction is not 0, so the curves intersect exactly when $A - C$ and $D - B$ have the same sign.

There are $6 \cdot 5 \cdot 4 \cdot 3 = 360$ ordered choices of four different values for (A, B, C, D). Swapping the values of B and D changes the sign of $D - B$ and does not change $A - C$. This swap pairs each choice in which the signs agree with exactly one choice in which they differ, so exactly half of the 360 choices, namely 180, give curves that intersect.

Each pair of curves is counted twice among these 180 choices: once as (A, B, C, D) and once with the two curves listed in the other order, (C, D, A, B). Listing the curves in the other order changes the signs of both $A - C$ and $D - B$, so it keeps a pair of intersecting curves intersecting. Therefore the number of ways is $180/2 = 90$.

Answer: (C) 90. □

Intermediate.

6. (2001 AIME I, Problem #6) A fair die is rolled four times. The probability that each of the final three rolls is at least as large as the roll preceding it may be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Strategy. A nondecreasing list of rolls is determined by how many rolls show each of the six values, so stars and bars counts the favorable lists.

Solution. The $6^4 = 1296$ lists of four rolls are equally likely, and the favorable ones are the lists $a_1 \le a_2 \le a_3 \le a_4$ with entries in $\{1, 2, \ldots, 6\}$. Such a list is determined by the numbers $x_1, x_2, \ldots, x_6$ of its entries equal to $1, 2, \ldots, 6$: writing x_1 ones, then x_2 twos, and so on, gives the list back. These numbers are nonnegative integers with

$$x_1 + x_2 + \cdots + x_6 = 4,$$

and every solution gives a nondecreasing list. By stars and bars, with 4 stars and 5 bars, there are $\binom{9}{4} = 126$ favorable lists. The probability is

$$\frac{126}{1296} = \frac{7}{72},$$

which is in lowest terms. Hence $m + n = 7 + 72 = 79$.

Answer: 79. □

7. (2019 AMC 10B, Problem #25) How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

(A) 55 **(B)** 60 **(C)** 65 **(D)** 70 **(E)** 75

Strategy. The 0s cut the sequence into blocks of 1s. Describe a sequence by the lengths of these blocks, and count the possible lists of lengths.

Solution. The sequence begins and ends with 0 and has no two consecutive 0s. So it consists of a 0, a block of 1s, a 0, a block of 1s, and so on, ending with a 0, where every block contains at least one 1. Since there are no three consecutive 1s, each block has length 1 or 2.

Suppose there are m zeros. Then there are $m - 1$ blocks, and if t of them have length 2, the blocks contain $(m - 1) + t$ ones in all. The length of the sequence is

$$m + (m - 1) + t = 19, \quad \text{so} \quad t = 20 - 2m.$$

The number t must satisfy $0 \le t \le m-1$. The condition $t \ge 0$ gives $m \le 10$, and $t \le m-1$ gives $20 - 2m \le m - 1$, that is, $m \ge 7$. For each m, a sequence is determined by choosing which t of the $m - 1$ blocks have length 2, so there are $\binom{m-1}{t}$ sequences:

Zeros m	Blocks of length 2, t	Sequences
7	6	$\binom{6}{6} = 1$
8	4	$\binom{7}{4} = 35$
9	2	$\binom{8}{2} = 28$
10	0	$\binom{9}{0} = 1$

The total is $1 + 35 + 28 + 1 = 65$.

Answer: (C) 65. □

8. (1986 AIME, Problem #7) The increasing sequence 1, 3, 4, 9, 10, 12, 13, ... consists of all those positive integers which are powers of 3 or sums of distinct powers of 3. Find the 100th term of this sequence.

Strategy. The terms are the positive integers whose base-3 digits are all 0 or 1. Reading those digits as a base-2 numeral matches the terms, in order, with the positive integers.

Solution. A sum of distinct powers of 3 has a base-3 representation with a 1 in the place of each power used and 0 in every other place. Since base-3 representations are unique, the terms of the sequence are exactly the positive integers whose base-3 digits are all 0 or 1.

Send each term to the positive integer with the same string of digits in base 2. For example, $12 = 110_3$ goes to $110_2 = 6$. The inverse reads the base-2 digits of a positive integer as base-3 digits, so this rule is a bijection between the terms and the positive integers.

The rule also keeps the order. In both bases, the place values to the right of a given place add up to less than that place value: $1 + 3 + \cdots + 3^{k-1} < 3^k$ and $1 + 2 + \cdots + 2^{k-1} < 2^k$. Hence, for strings of 0s and 1s beginning with 1, read in either base, a longer string gives a larger number, and of two strings of the same length, the larger number has a 1 where the strings first differ. So the kth term goes to k for every k, and the 100th term has the same digits as 100 in base 2. Since $100 = 64 + 32 + 4 = 1100100_2$, the 100th term is

$$1100100_3 = 3^6 + 3^5 + 3^2 = 729 + 243 + 9 = 981.$$

Answer: 981. □

9. (2021 Fall AMC 12A, Problem #18) Each of 20 balls is tossed independently and at random into one of 5 bins. Let p be the probability that some bin ends up with 3 balls, another with 5 balls, and the other three with 4 balls each. Let q be the probability that every bin ends up with 4 balls. What is $\frac{p}{q}$?

(A) 1 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 16

Strategy. An outcome with given numbers of balls in the bins is an arrangement of a word with repeated letters. Count both events this way and divide; the factor 20! cancels.

Solution. Number the balls 1 through 20 and the bins 1 through 5. Record an outcome as the word of length 20 whose kth letter is the bin of ball k. All 5^{20} words are equally likely. The outcomes in which bins $1, 2, \ldots, 5$ receive $c_1, c_2, \ldots, c_5$ balls are the arrangements of a word with c_i copies of the letter i, and there are

$$\frac{20!}{c_1!\, c_2!\, c_3!\, c_4!\, c_5!}$$

of them.

For q, every c_i equals 4, which gives $20!/(4!)^5$ outcomes. For p, choose the bin with 3 balls in 5 ways and then the bin with 5 balls in 4 ways. Different choices give different numbers of balls in the bins, so their outcomes do not overlap, and each of the 20 choices gives $20!/\big(3!\,5!\,(4!)^3\big)$ outcomes. The two probabilities have the same denominator 5^{20}, so

$$\frac{p}{q} = \frac{20 \cdot 20!/\big(3!\,5!\,(4!)^3\big)}{20!/(4!)^5} = \frac{20 \cdot (4!)^2}{3!\,5!} = \frac{20 \cdot 576}{720} = 16.$$

Answer: (E) 16. □

10. (2010 AMC 12A, Problem #16) Bernardo randomly picks 3 distinct numbers from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8\}$$

and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

Strategy. Split according to whether Bernardo picks 9. If he does not, exchanging the two players' picks pairs his wins with his losses, and the numbers tie only when the two sets of picks are equal.

Solution. A number whose three different digits are in descending order is determined by its set of digits, and different sets give different numbers. So Bernardo picks one of the $\binom{9}{3} = 84$ three-element subsets of $\{1, 2, \ldots, 9\}$ and Silvia one of the $\binom{8}{3} = 56$ three-element subsets of $\{1, 2, \ldots, 8\}$, each equally likely and independently. The $84 \cdot 56 = 4704$ pairs of sets are equally likely.

If Bernardo's set contains 9, his number begins with 9 and Silvia's begins with a smaller digit, so his number is larger. He has $\binom{8}{2} = 28$ such sets, which give $28 \cdot 56 = 1568$ winning pairs.

Otherwise both sets come from the same 56 subsets of $\{1, 2, \ldots, 8\}$, which gives $56^2 = 3136$ pairs. Exchanging the two sets of such a pair is its own inverse. It turns a pair in which Bernardo's number is larger into a pair in which Silvia's number is larger, and conversely. The numbers are equal exactly when the sets are equal, which happens in 56 pairs. So the remaining $3136 - 56 = 3080$ pairs split into two halves of the same size, and Bernardo wins in 1540 of them. The probability is

$$\frac{1568 + 1540}{4704} = \frac{3108}{4704} = \frac{37}{56}.$$

Answer: (B) $\frac{37}{56}$. □

Advanced.

11. (2003 AMC 12A, Problem #22) Objects A and B move simultaneously in the coordinate plane via a sequence of steps, each of length one. Object A starts at $(0, 0)$ and each of its steps is either right or up, both equally likely. Object B starts at $(5, 7)$ and each of its steps is either left or down, both equally likely. Which of the following is closest to the probability that the objects meet?

(A) 0.10 **(B)** 0.15 **(C)** 0.20 **(D)** 0.25 **(E)** 0.30

Strategy. Each step raises the coordinate sum of A by 1 and lowers that of B by 1, so the objects can meet only after 6 steps each. Encode the first 6 steps of each object as a word and count the words that lead to each meeting point.

Solution. Measure time in steps. At time t, even in the middle of a step, the coordinates of A add up to t and the coordinates of B add up to $12-t$. So the objects can be at the same point only at time 6, when each has taken 6 steps.

Encode the first 6 steps of A by a word in the letters R and U. Then A is at $(i, 6-i)$ at time 6 exactly when its word has i letters R, which happens for $\binom{6}{i}$ of the $2^6 = 64$ equally likely words. Encode the first 6 steps of B by a word in the letters L and D. Then B is at $(5-j, 1+j)$ at time 6 when its word has j letters L, and this point is $(i, 6-i)$ exactly when $j = 5-i$. So the objects can meet only at the points $(i, 6-i)$ with $0 \le i \le 5$, and B reaches $(i, 6-i)$ for $\binom{6}{5-i}$ of its 64 words. The objects move independently, and different meeting points give cases that do not overlap, so the probability that the objects meet is

$$\frac{1}{64^2}\sum_{i=0}^{5}\binom{6}{i}\binom{6}{5-i} = \frac{6+90+300+300+90+6}{4096} = \frac{792}{4096} \approx 0.193.$$

The total 792 equals $\binom{12}{5}$, because splitting a path of right and up steps from $(0,0)$ to $(5,7)$ at its point on the line $x+y=6$ gives a word for A and, from the second half traversed backward, a word for B that ends at the same point. Conversely, when the two words end at the same point, the first 6 steps of A followed by those of B traversed backward form such a path, and splitting it recovers the two words. The closest choice is 0.20.

Answer: (C) 0.20. □

12. (2008 AMC 12B, Problem #22) A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers choose their spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

(A) $\frac{11}{20}$ **(B)** $\frac{4}{7}$ **(C)** $\frac{81}{140}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{17}{28}$

Strategy. After the twelve cars park, the four empty spaces form a random four-element set. Auntie Em cannot park exactly when this set contains no two adjacent spaces, and such sets are separated selections.

Solution. Number the spaces 1 through 16. The drivers choose 12 different spaces one after another, and every ordered choice has the same probability $\frac{1}{16}\cdot\frac{1}{15}\cdots\frac{1}{5}$. Each set of 12 occupied spaces comes from 12! orders, so all sets of occupied spaces are equally likely. Equivalently, the set of 4 empty spaces is equally likely to be any of the $\binom{16}{4} = 1820$ four-element subsets of $\{1, 2, \ldots, 16\}$.

Auntie Em can park exactly when two of the empty spaces are adjacent. By the count of separated selections with $d = 2$, the number of four-element subsets of $\{1, 2, \ldots, 16\}$ with no two consecutive numbers is $\binom{16-3}{4} = \binom{13}{4} = 715$. The probability that she can park is therefore

$$1 - \frac{715}{1820} = 1 - \frac{11}{28} = \frac{17}{28}.$$

Answer: (E) $\frac{17}{28}$. □

13. (1986 AIME, Problem #13) In a sequence of coin tosses, one can keep a record of instances in which a tail is immediately followed by a head, a head is immediately followed by a head, and so on. We denote these by TH, HH, and so on. For example, in the sequence TTTHHTHTTTHHTTH of 15 coin tosses, there are two HH, three HT, four TH, and five TT subsequences. How many different sequences of 15 coin tosses contain exactly two HH, three HT, four TH, and five TT subsequences?

Strategy. The pairs HT and TH mark the changes between runs, so they fix the pattern of runs. The pairs HH and TT then fix the total length of the runs of heads and of the runs of tails.

Solution. Split a sequence of tosses into runs. The pairs HT and TH are exactly the places where one run ends and the next begins, so a sequence with the required counts has $3+4=7$ changes and 8 runs. The runs alternate between heads and tails, so the changes alternate between TH and HT. With four TH and three HT, the changes must be TH, HT, TH, HT, TH, HT, TH in this order. So the sequence begins with tails and ends with heads, and it has 4 runs of tails and 4 runs of heads.

A run of length L contains $L-1$ pairs of equal neighbors. Hence the lengths of the 4 runs of heads add up to $2+4=6$, and the lengths of the 4 runs of tails add up to $5+4=9$.

Conversely, take positive integers $h_1,\dots,h_4$ with sum 6 and $t_1,\dots,t_4$ with sum 9, and write t_1 tails, h_1 heads, t_2 tails, and so on, ending with h_4 heads. The result has 15 tosses, four TH and three HT, and its runs contain $6-4=2$ pairs HH and $9-4=5$ pairs TT. A sequence determines its run lengths, so by stars and bars for positive solutions the number of sequences is

$$\binom{6-1}{4-1}\binom{9-1}{4-1}=\binom{5}{3}\binom{8}{3}=10\cdot 56=560.$$

Answer: 560. □

14. (2010 AIME II, Problem #8) Let N be the number of ordered pairs of nonempty sets $\mathcal{A}$ and $\mathcal{B}$ that have the following properties:

- $\mathcal{A}\cup\mathcal{B}=\{1,2,3,4,5,6,7,8,9,10,11,12\}$,
- $\mathcal{A}\cap\mathcal{B}=\emptyset$,
- the number of elements of $\mathcal{A}$ is not an element of $\mathcal{A}$,
- the number of elements of $\mathcal{B}$ is not an element of $\mathcal{B}$.

Find N.

Strategy. If $\mathcal{A}$ has k elements, the last two conditions force k into $\mathcal{B}$ and $12-k$ into $\mathcal{A}$. Count the sets $\mathcal{A}$ of each size.

Solution. By the first two conditions, $\mathcal{B}$ is the complement of $\mathcal{A}$ in $\{1,2,\dots,12\}$, so a pair is determined by $\mathcal{A}$. Let $\mathcal{A}$ have k elements. Then $\mathcal{B}$ has $12-k$ elements, and $1\le k\le 11$ because both sets are nonempty. The third condition says that $k\notin\mathcal{A}$, that is, $k\in\mathcal{B}$. The fourth says that $12-k\notin\mathcal{B}$, that is, $12-k\in\mathcal{A}$.

For $k=6$ these conditions put 6 in both sets, which is impossible. For $k\neq 6$ the numbers k and $12-k$ are different, and $\mathcal{A}$ consists of $12-k$ together with $k-1$ of the 10 numbers other than k and $12-k$. Each such choice satisfies all four conditions, so there are $\binom{10}{k-1}$ pairs with $|\mathcal{A}|=k$.

Write $j=k-1$, which runs over $0,1,\dots,10$ except 5. Then N is the sum of $\binom{10}{j}$ over these values of j. For each j, the number $\binom{10}{j}$ counts the j-element subsets of a fixed 10-element set, so by the Addition Principle N is the number of its subsets of every size

other than 5. A 10-element set has $2^{10} = 1024$ subsets, and $\binom{10}{5} = 252$ of them have 5 elements. Therefore

$$N = 1024 - 252 = 772.$$

Answer: 772. □

15. A one-way street has 6 parking spaces in a row, numbered 1 through 6 in the direction of traffic. Four cars arrive one at a time, and each driver has a preferred space, one of the six. A driver goes to the preferred space and parks in the first empty space at or after it; a driver who finds no such space leaves without parking. Different drivers may prefer the same space. For how many of the 6^4 lists of preferences, one entry for each car in order of arrival, do all four cars park?

Strategy. Add a seventh space and close the street into a circle, where every list of preferences lets all four cars park and leaves three spaces empty. Rotating a list rotates its empty spaces, so exactly three of its seven rotations leave the new space empty.

Solution. Add a space 7 after space 6 and join the end of the street to its beginning, so that the spaces $1, 2, \ldots, 7$ lie around a circle in this order and space 1 follows space 7. Let each driver prefer any of the 7 spaces and drive around the circle from the preferred space to the first empty one. There are only 4 cars, so every driver parks and exactly 3 spaces stay empty. There are 7^4 such lists of preferences.

For $r = 1, 2, \ldots, 6$, the rotation by r adds r to every preference and subtracts 7 from any result above 7. Every driver of the rotated list moves exactly as before, shifted r spaces around the circle, so the empty spaces are shifted by r as well. The 7 rotations of a list, the list itself included, are different, because its first preference takes 7 different values. A rotation of a rotation is again a rotation, so the 7^4 lists split into $7^4/7 = 7^3$ groups of 7 lists that are rotations of one another. Within a group, each of the 3 empty spaces of one list is shifted onto space 7 by exactly one rotation, and different empty spaces by different rotations. So exactly 3 lists of each group leave space 7 empty, and the number of such lists is

$$3 \cdot 7^3 = 1029.$$

Finally, a list on the circle leaves space 7 empty exactly when it is a list for the street in which all four cars park. If space 7 stays empty, no driver prefers it and no driver passes it, since a driver who reaches an empty space parks there. So every preference is one of $1, 2, \ldots, 6$, and every driver parks in the first empty space at or after the preferred one, just as on the street. Conversely, if all four cars park on the street, then on the circle every driver finds the same space before reaching space 7, which stays empty. Hence the answer is 1029.

Answer: 1029. □

2. Answer Key

Problem	Letter	Value
1	D	39
2	D	28
3		300
4	B	20
5	C	90
6		79
7	C	65
8		981
9	E	16
10	B	$\frac{37}{56}$
11	C	0.20
12	E	$\frac{17}{28}$
13		560
14		772
15		1029