# SYMMETRIC SUMS, NEWTON'S SUMS, AND ROOT TRANSFORMATIONS FOR AMC 12: PRACTICE SET

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## 1. Problems

**Beginner.**

**1.** (1996 AHSME, Problem #23) The sum of the lengths of the twelve edges of a rectangular box is 140, and the distance from one corner of the box to the farthest corner is 21. The total surface area of the box is

**(A)** 776 **(B)** 784 **(C)** 798 **(D)** 800 **(E)** 812

**2.** (2007 AMC 10A, Problem #20) Suppose that the number $a$ satisfies the equation $4 = a + a^{-1}$. What is the value of $a^4 + a^{-4}$?

**(A)** 164 **(B)** 172 **(C)** 192 **(D)** 194 **(E)** 212

**3.** (2005 AMC 12B, Problem #12) The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $x^2 + px + m = 0$, and none of $m$, $n$, and $p$ is zero. What is the value of $\dfrac{n}{p}$?

**(A)** 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

**4.** (1960 AHSME, Problem #31) For $x^2 + 2x + 5$ to be a factor of $x^4 + px^2 + q$, the values of $p$ and $q$ must be, respectively:

**(A)** $-2$, 5 **(B)** 5, 25 **(C)** 10, 20 **(D)** 6, 25 **(E)** 14, 25

**5.** (1983 AHSME, Problem #20) If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 - px + q = 0$, and $\cot\alpha$ and $\cot\beta$ are the roots of $x^2 - rx + s = 0$, then $rs$ is necessarily

**(A)** $pq$ **(B)** $\dfrac{1}{pq}$ **(C)** $\dfrac{p}{q^2}$ **(D)** $\dfrac{q}{p^2}$ **(E)** $\dfrac{p}{q}$

**Intermediate.**

**6.** (2014 AMC 12B, Problem #16) Let $P$ be a cubic polynomial with $P(0) = k$, $P(1) = 2k$, and $P(-1) = 3k$. What is $P(2) + P(-2)$?

**(A)** 0 **(B)** $k$ **(C)** $6k$ **(D)** $7k$ **(E)** $14k$

**7.** (2003 AIME I, Problem #4) Given that $\log_{10}\sin x + \log_{10}\cos x = -1$ and that $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, find $n$.

**8.** (1977 AHSME, Problem #23) If the solutions of the equation $x^2 + px + q = 0$ are the cubes of the solutions of the equation $x^2 + mx + n = 0$, then

**(A)** $p = m^3 + 3mn$ **(B)** $p = m^3 - 3mn$ **(C)** $p + q = m^3$ **(D)** $\left(\dfrac{m}{n}\right)^3 = \dfrac{p}{q}$

**(E)** none of these

**9.** (1990 AHSME, Problem #30) If $R_n = \frac{1}{2}(a^n + b^n)$, where $a = 3 + 2\sqrt{2}$, $b = 3 - 2\sqrt{2}$, and $n = 0, 1, 2, \ldots$, then $R_{12345}$ is an integer. Its units digit is

**(A)** 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

**10.** (2013 AIME I, Problem #5) The real root of the equation $8x^3 - 3x^2 - 3x - 1 = 0$ can be written in the form $\dfrac{\sqrt[3]{a} + \sqrt[3]{b} + 1}{c}$, where $a$, $b$, and $c$ are positive integers. Find $a + b + c$.

**Advanced.**

**11.** (1999 AHSME, Problem #30) The number of ordered pairs of integers $(m, n)$ for which $mn \geq 0$ and

$$m^3 + n^3 + 99mn = 33^3$$

is equal to

**(A)** 2 **(B)** 3 **(C)** 33 **(D)** 35 **(E)** 99

**12.** (2014 AIME I, Problem #9) Let $x_1 < x_2 < x_3$ be the three real roots of the equation $\sqrt{2014}\,x^3 - 4029x^2 + 2 = 0$. Find $x_2(x_1 + x_3)$.

**13.** (2019 AIME I, Problem #10) For distinct complex numbers $z_1$, $z_2$, ..., $z_{673}$, the polynomial

$$(x - z_1)^3(x - z_2)^3 \cdots (x - z_{673})^3$$

can be expressed as $x^{2019} + 20x^{2018} + 19x^{2017} + g(x)$, where $g(x)$ is a polynomial with complex coefficients and with degree at most 2016. The value of

$$\left| \sum_{1 \leq j < k \leq 673} z_j z_k \right|$$

can be expressed in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

**14.** (2024 AIME II, Problem #11) Find the number of triples of nonnegative integers $(a, b, c)$ satisfying $a + b + c = 300$ and

$$a^2b + a^2c + b^2a + b^2c + c^2a + c^2b = 6{,}000{,}000.$$

**15.** (2026 AIME II, Problem #11) Find the greatest integer $n$ such that the cubic polynomial

$$x^3 - \frac{n}{6}x^2 + (n - 11)x - 400$$

has roots $\alpha^2$, $\beta^2$, and $\gamma^2$, where $\alpha$, $\beta$, and $\gamma$ are complex numbers, and there are exactly seven different possible values for $\alpha + \beta + \gamma$.

## 2. Answer Key

| Problem | Letter | Value |
|---|---|---|
| 1 | B | 784 |
| 2 | D | 194 |
| 3 | D | 8 |
| 4 | D | 6, 25 |
| 5 | C | $\frac{p}{q^2}$ |
| 6 | E | $14k$ |
| 7 | | 12 |
| 8 | B | $p = m^3 - 3mn$ |
| 9 | E | 9 |
| 10 | | 98 |
| 11 | D | 35 |
| 12 | | 2 |
| 13 | | 352 |
| 14 | | 601 |
| 15 | | 132 |