# POLYNOMIALS: FACTOR THEOREM, RATIONAL ROOTS, AND VIETA FOR AMC 12: PRACTICE SET

ANDY JO

## Contents

1. Problems 2
2. Answer Key 4

## 1. Problems

**Beginner.**

**1.** (1982 AHSME, Problem #1) When the polynomial $x^3 - 2$ is divided by the polynomial $x^2 - 2$, the remainder is

**(A)** $2$ **(B)** $-2$ **(C)** $-2x - 2$ **(D)** $2x + 2$ **(E)** $2x - 2$

**2.** (2018 AMC 12A, Problem #5) What is the sum of all possible values of $k$ for which the polynomials $x^2 - 3x + 2$ and $x^2 - 5x + k$ have a root in common?

**(A)** $3$ **(B)** $4$ **(C)** $5$ **(D)** $6$ **(E)** $10$

**3.** (1954 AHSME, Problem #20) The equation $x^3 + 6x^2 + 11x + 6 = 0$ has:

**(A)** no negative real roots **(B)** no positive real roots **(C)** no real roots
**(D)** 1 positive and 2 negative roots **(E)** 2 positive and 1 negative root

**4.** (1965 AHSME, Problem #27) When $y^2 + my + 2$ is divided by $y - 1$ the quotient is $f(y)$ and the remainder is $R_1$. When $y^2 + my + 2$ is divided by $y + 1$ the quotient is $g(y)$ and the remainder is $R_2$. If $R_1 = R_2$, then $m$ is:

**(A)** $0$ **(B)** $1$ **(C)** $2$ **(D)** $-1$ **(E)** an undetermined constant

**5.** (1956 AHSME, Problem #17) The fraction $\dfrac{5x - 11}{2x^2 + x - 6}$ was obtained by adding the two fractions $\dfrac{A}{x + 2}$ and $\dfrac{B}{2x - 3}$. The values of $A$ and $B$ must be, respectively:

**(A)** $5x$, $-11$ **(B)** $-11$, $5x$ **(C)** $-1$, $3$ **(D)** $3$, $-1$ **(E)** $5$, $-11$

**Intermediate.**

**6.** (2020 AMC 10A, Problem #17) Define

$$P(x) = (x - 1^2)(x - 2^2) \cdots (x - 100^2).$$

How many integers $n$ are there such that $P(n) \leq 0$?

**(A)** $4900$ **(B)** $4950$ **(C)** $5000$ **(D)** $5050$ **(E)** $5100$

**7.** (2022 AIME I, Problem #1) Quadratic polynomials $P(x)$ and $Q(x)$ have leading coefficients of $2$ and $-2$, respectively. The graphs of both polynomials pass through the two points $(16, 54)$ and $(20, 53)$. Find $P(0) + Q(0)$.

**8.** (1983 AHSME, Problem #18) Let $f$ be a polynomial function such that, for all real $x$,

$$f(x^2 + 1) = x^4 + 5x^2 + 3.$$

For all real $x$, $f(x^2 - 1)$ is

**(A)** $x^4 + 5x^2 + 1$ **(B)** $x^4 + x^2 - 3$ **(C)** $x^4 - 5x^2 + 1$ **(D)** $x^4 + x^2 + 3$
**(E)** none of these

**9.** (2003 AMC 12A, Problem #21) The graph of the polynomial

$$P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$$

has five distinct $x$-intercepts, one of which is at $(0, 0)$. Which of the following coefficients cannot be zero?

**(A)** $a$ **(B)** $b$ **(C)** $c$ **(D)** $d$ **(E)** $e$

**10.** The polynomial $x^4 + ax^3 + bx^2 + cx + d$, where $a$, $b$, $c$, and $d$ are rational numbers, has $1 + \sqrt{2}$ as a double root. What is $a + b + c + d$?
**(A)** $-4$ **(B)** $-1$ **(C)** 1 **(D)** 3 **(E)** 4

**Advanced.**

**11.** (2010 AIME I, Problem #6) Let $P(x)$ be a quadratic polynomial with real coefficients satisfying

$$x^2 - 2x + 2 \leq P(x) \leq 2x^2 - 4x + 3$$

for all real numbers $x$, and suppose $P(11) = 181$. Find $P(16)$.

**12.** (2005 AIME I, Problem #8) The equation

$$2^{333x-2} + 2^{111x+2} = 2^{222x+1} + 1$$

has three real roots. Given that their sum is $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers, find $m + n$.

**13.** (2015 AIME II, Problem #6) Steve says to Jon, "I am thinking of a polynomial whose roots are all positive integers. The polynomial has the form $P(x) = 2x^3 - 2ax^2 + (a^2 - 81)x - c$ for some positive integers $a$ and $c$. Can you tell me the values of $a$ and $c$?"
After some calculations, Jon says, "There is more than one such polynomial."
Steve says, "You're right. Here is the value of $a$." He writes down a positive integer and asks, "Can you tell me the value of $c$?"
Jon says, "There are still two possible values of $c$."
Find the sum of the two possible values of $c$.

**14.** (2018 AIME II, Problem #6) A real number $a$ is chosen randomly and uniformly from the interval $[-20, 18]$. The probability that the roots of the polynomial

$$x^4 + 2ax^3 + (2a - 2)x^2 + (-4a + 3)x - 2$$

are all real can be written in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

**15.** (2005 AMC 12A, Problem #24) Let $P(x) = (x-1)(x-2)(x-3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x) \cdot R(x)$?
**(A)** 19 **(B)** 22 **(C)** 24 **(D)** 27 **(E)** 32

## 2. Answer Key

| Problem | Letter | Value |
|---|---|---|
| 1 | E | $2x - 2$ |
| 2 | E | 10 |
| 3 | B | no positive real roots |
| 4 | A | 0 |
| 5 | D | $3, -1$ |
| 6 | E | 5100 |
| 7 | | 116 |
| 8 | B | $x^4 + x^2 - 3$ |
| 9 | D | $d$ |
| 10 | D | 3 |
| 11 | | 406 |
| 12 | | 113 |
| 13 | | 440 |
| 14 | | 37 |
| 15 | B | 22 |