LOGARITHMS AND EXPONENTIAL EQUATIONS FOR AMC 12: PRACTICE SET

ANDY JO

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1. Problems

Beginner.

1. (2000 AMC 12, Problem #7) How many positive integers b have the property that $\log_b 729$ is a positive integer?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

2. (1958 AHSME, Problem #20) If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals:

(A) $5\sqrt{5}$ **(B)** $\sqrt{5}$ **(C)** $25\sqrt{5}$ **(D)** 125 **(E)** 25

3. (1982 AHSME, Problem #17) How many real numbers x satisfy the equation $3^{2x+2} - 3^{x+3} - 3^x + 3 = 0$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. (2018 AMC 12B, Problem #7) What is the value of

$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

(A) 3 **(B)** $3\log_7 23$ **(C)** 6 **(D)** 9 **(E)** 10

5. (2023 AMC 12B, Problem #7) For how many integers n does the expression

$$\sqrt{\frac{\log(n^2) - (\log n)^2}{\log n - 3}}$$

represent a real number, where log denotes the base 10 logarithm?

(A) 900 **(B)** 3 **(C)** 902 **(D)** 2 **(E)** 901

Intermediate.

6. (2022 AMC 12A, Problem #11) What is the product of all real numbers x such that the distance on the number line between $\log_6 x$ and $\log_6 9$ is twice the distance on the number line between $\log_6 10$ and 1?

(A) 10 **(B)** 18 **(C)** 25 **(D)** 36 **(E)** 81

7. (2022 AMC 12A, Problem #14) What is the value of

$$(\log 5)^3 + (\log 20)^3 + (\log 8)(\log 0.25)$$

where log denotes the base-ten logarithm?

(A) $\frac{3}{2}$ **(B)** $\frac{7}{4}$ **(C)** 2 **(D)** $\frac{9}{4}$ **(E)** 3

8. (2021 AMC 12A, Problem #14) What is the value of

$$\left(\sum_{k=1}^{20} \log_{5^k} 3^{k^2}\right) \cdot \left(\sum_{k=1}^{100} \log_{9^k} 25^k\right)?$$

(A) 21 **(B)** $100\log_5 3$ **(C)** $200\log_3 5$ **(D)** 2,200 **(E)** 21,000

9. (1968 AHSME, Problem #29) Given the three numbers x, $y = x^x$, $z = x^{(x^x)}$, with $0.9 < x < 1.0$. Arranged in order of increasing magnitude, they are:

(A) x, z, y **(B)** x, y, z **(C)** y, x, z **(D)** y, z, x **(E)** z, x, y

10. (2009 AIME II, Problem #2) Suppose that a, b, and c are positive real numbers such that $a^{\log_3 7} = 27$, $b^{\log_7 11} = 49$, and $c^{\log_{11} 25} = \sqrt{11}$. Find

$$a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}.$$

Advanced.

11. (2017 AMC 12A, Problem #20) How many ordered pairs (a, b) such that a is a positive real number and b is an integer between 2 and 200, inclusive, satisfy the equation $(\log_b a)^{2017} = \log_b(a^{2017})$?

(A) 198 **(B)** 199 **(C)** 398 **(D)** 399 **(E)** 597

12. (2022 AIME II, Problem #4) There is a positive real number x not equal to either $\frac{1}{20}$ or $\frac{1}{2}$ such that

$$\log_{20x}(22x) = \log_{2x}(202x).$$

The value $\log_{20x}(22x)$ can be written as $\log_{10}\left(\frac{m}{n}\right)$, where m and n are relatively prime positive integers. Find $m + n$.

13. (1988 AHSME, Problem #26) Suppose that p and q are positive numbers for which

$$\begin{aligned}\log_9(p) &= \log_{12}(q)\\ &= \log_{16}(p+q).\end{aligned}$$

What is the value of $\frac{q}{p}$?

(A) $\frac{4}{3}$ **(B)** $\frac{1}{2}(1+\sqrt{3})$ **(C)** $\frac{8}{5}$ **(D)** $\frac{1}{2}(1+\sqrt{5})$ **(E)** $\frac{16}{9}$

14. (2009 AMC 12A, Problem #24) The tower function of twos is defined recursively as follows: $T(1) = 2$ and $T(n+1) = 2^{T(n)}$ for $n \geq 1$. Let $A = (T(2009))^{T(2009)}$ and $B = (T(2009))^A$. What is the largest integer k such that

$$\underbrace{\log_2 \log_2 \log_2 \ldots \log_2}_{k} B$$

is defined?

(A) 2009 **(B)** 2010 **(C)** 2011 **(D)** 2012 **(E)** 2013

15. (2017 AIME II, Problem #7) Find the number of integer values of k in the closed interval $[-500, 500]$ for which the equation $\log(kx) = 2\log(x+2)$ has exactly one real solution.

2. Answer Key

Problem	Letter	Value
1	E	4
2	C	$25\sqrt{5}$
3	C	2
4	C	6
5	E	901
6	E	81
7	C	2
8	E	21,000
9	A	x, z, y
10		469
11	E	597
12		112
13	D	$\frac{1}{2}(1+\sqrt{5})$
14	E	2013
15		501