SIMILAR TRIANGLES AND AREA RATIOS FOR AMC 10: PRACTICE SET

ANDY JO

Contents

1. Problems 2
2. Answer Key 5

1. Problems

Beginner.

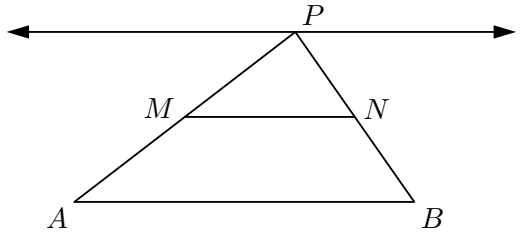
1. (1988 AHSME, Problem #2) Triangles ABC and XYZ are similar, with A corresponding to X and B to Y. If $AB = 3$, $BC = 4$, and $XY = 5$, then YZ is

(A) $3\frac{3}{4}$ **(B)** 6 **(C)** $6\frac{1}{4}$ **(D)** $6\frac{2}{3}$ **(E)** 8

2. (2016 AMC 10B, Problem #10) A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

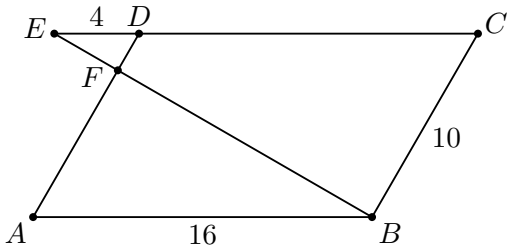
(A) 14.0 **(B)** 16.0 **(C)** 20.0 **(D)** 33.3 **(E)** 55.6

3. (2000 AMC 10, Problem #5) Points M and N are the midpoints of sides PA and PB of $\triangle PAB$. As P moves along a line that is parallel to side AB, how many of the four quantities listed below change?
(a) the length of the segment MN; (b) the perimeter of $\triangle PAB$; (c) the area of $\triangle PAB$; (d) the area of trapezoid $ABNM$.



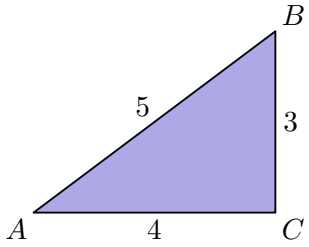
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. (1990 AHSME, Problem #4) Let $ABCD$ be a parallelogram with $\angle ABC = 120°$, $AB = 16$, and $BC = 10$. Extend $\overline{CD}$ through D to E so that $DE = 4$. If $\overline{BE}$ intersects $\overline{AD}$ at F, then FD is closest to



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

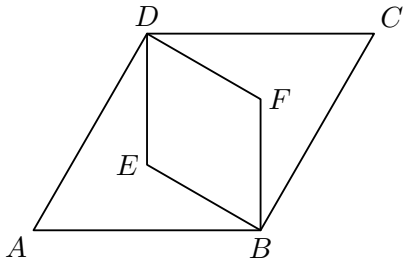
5. (2018 AMC 10A, Problem #13) A paper triangle with sides of lengths 3, 4, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



(A) $1+\frac{1}{2}\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{15}{8}$ **(E)** 2

Intermediate.

6. (2006 AMC 10B, Problem #15) Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 24, and $\angle BAD = 60^\circ$. What is the area of rhombus $BFDE$?



(A) 6 **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** 9 **(E)** $6\sqrt{3}$

7. (2022 AMC 10A, Problem #13) Let $\triangle ABC$ be a scalene triangle. Point P lies on $\overline{BC}$ so that $\overline{AP}$ bisects $\angle BAC$. The line through B perpendicular to $\overline{AP}$ intersects the line through A parallel to $\overline{BC}$ at point D. Suppose $BP = 2$ and $PC = 3$. What is AD?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

8. (1955 AHSME, Problem #31) An equilateral triangle whose side is 2 is divided into a triangle and a trapezoid by a line drawn parallel to one of its sides. If the area of the trapezoid equals one-half of the area of the original triangle, the length of the median of the trapezoid is:

(A) $\frac{\sqrt{6}}{2}$ **(B)** $\sqrt{2}$ **(C)** $2+\sqrt{2}$ **(D)** $\frac{2+\sqrt{2}}{2}$ **(E)** $\frac{2\sqrt{3}-\sqrt{6}}{2}$

9. (1956 AHSME, Problem #38) In a right triangle with sides a and b, and hypotenuse c, the altitude drawn on the hypotenuse is x. Then:

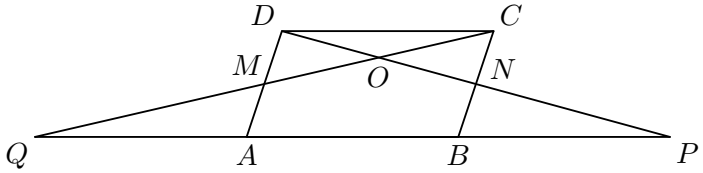
(A) $ab = x^2$ **(B)** $\frac{1}{a}+\frac{1}{b}=\frac{1}{x}$ **(C)** $a^2+b^2=2x^2$ **(D)** $\frac{1}{x^2}=\frac{1}{a^2}+\frac{1}{b^2}$

(E) $\frac{1}{x}=\frac{b}{a}$

10. (2017 AMC 10B, Problem #19) Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3 \cdot AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3 \cdot BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3 \cdot CA$.
What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?
(A) $9:1$ **(B)** $16:1$ **(C)** $25:1$ **(D)** $36:1$ **(E)** $37:1$

Advanced.

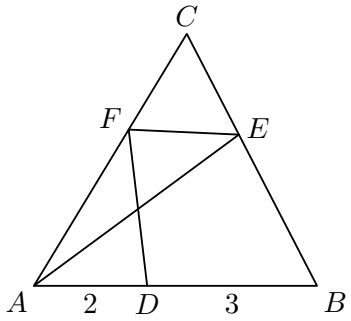
11. (1974 AHSME, Problem #25) In parallelogram $ABCD$ of the accompanying diagram, line DP is drawn bisecting BC at N and meeting AB (extended) at P. From vertex C, line CQ is drawn bisecting side AD at M and meeting AB (extended) at Q. Lines DP and CQ meet at O. If the area of parallelogram $ABCD$ is k, then the area of triangle QPO is equal to



(A) k **(B)** $\frac{6k}{5}$ **(C)** $\frac{9k}{8}$ **(D)** $\frac{5k}{4}$ **(E)** $2k$

12. (2011 AIME II, Problem #4) In triangle ABC, $AB = \frac{20}{11}AC$. The angle bisector of angle A intersects $\overline{BC}$ at point D, and point M is the midpoint of $\overline{AD}$. Let P be the point of intersection of $\overline{AC}$ and line BM. The ratio of CP to PA can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

13. (1983 AHSME, Problem #28) Triangle ABC in the figure has area 10. Points D, E and F, all distinct from A, B and C, are on sides AB, BC and CA respectively, and $AD = 2$, $DB = 3$. If triangle ABE and quadrilateral $DBEF$ have equal areas, then that area is



(A) 4 **(B)** 5 **(C)** 6 **(D)** $\frac{5}{3}\sqrt{10}$ **(E)** not uniquely determined

14. (1986 AIME, Problem #9) In $\triangle ABC$, $AB = 425$, $BC = 450$, and $AC = 510$. An interior point P is drawn, and segments are drawn through P parallel to the sides of the triangle. If these three segments have equal length d, find d.

15. (2019 AIME I, Problem #6) In convex quadrilateral $KLMN$, side $\overline{MN}$ is perpendicular to diagonal $\overline{KM}$, side $\overline{KL}$ is perpendicular to diagonal $\overline{LN}$, $MN = 65$, and $KL = 28$. The line through L perpendicular to side $\overline{KN}$ intersects diagonal $\overline{KM}$ at O with $KO = 8$. Find MO.

2. Answer Key

Problem	Letter	Value
1	D	$6\frac{2}{3}$
2	D	33.3
3	B	1
4	B	2
5	D	$\frac{15}{8}$
6	C	8
7	C	10
8	D	$\frac{2+\sqrt{2}}{2}$
9	D	$\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{b^2}$
10	E	$37:1$
11	C	$\frac{9k}{8}$
12		51
13	C	6
14		306
15		90