SIMILAR TRIANGLES AND AREA RATIOS FOR AMC 10: PRACTICE SET SOLUTIONS

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1. Solutions

Beginner.

1. (1988 AHSME, Problem #2) Triangles ABC and XYZ are similar, with A corresponding to X and B to Y. If $AB = 3$, $BC = 4$, and $XY = 5$, then YZ is

(A) $3\frac{3}{4}$ **(B)** 6 **(C)** $6\frac{1}{4}$ **(D)** $6\frac{2}{3}$ **(E)** 8

Strategy. The correspondence also sends C to Z, so the side YZ corresponds to BC. The pair AB and XY gives the ratio of similarity.

Solution. Since A corresponds to X and B to Y, the third vertex C corresponds to Z, and $\triangle ABC \sim \triangle XYZ$ with the letters in this order. The side XY corresponds to AB, so the ratio of similarity is

$$k = \frac{XY}{AB} = \frac{5}{3}.$$

The side YZ corresponds to BC, so

$$YZ = k \cdot BC = \frac{5}{3} \cdot 4 = \frac{20}{3} = 6\tfrac{2}{3}.$$

The lengths CA and ZX are not needed.

Answer: (D) $6\frac{2}{3}$. □

2. (2016 AMC 10B, Problem #10) A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

(A) 14.0 **(B)** 16.0 **(C)** 20.0 **(D)** 33.3 **(E)** 55.6

Strategy. The weights of the two pieces are in the ratio of their areas. Any two equilateral triangles are similar, so their areas are in the ratio of the squares of their sides.

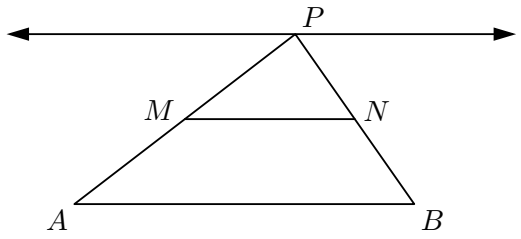
Solution. The two pieces are cut from the same wood with the same thickness, so the weight of each piece is proportional to its area. Each angle of an equilateral triangle is $60°$, so the two triangles are similar by AA, with ratio of similarity $\frac{5}{3}$. By the theorem on areas of similar triangles, the second piece has $\left(\frac{5}{3}\right)^2 = \frac{25}{9}$ times the area of the first, so it weighs

$$12 \cdot \frac{25}{9} = \frac{100}{3} = 33.33\ldots$$

ounces. The closest choice is 33.3.

Answer: (D) 33.3. □

3. (2000 AMC 10, Problem #5) Points M and N are the midpoints of sides PA and PB of $\triangle PAB$. As P moves along a line that is parallel to side AB, how many of the four quantities listed below change?
(a) the length of the segment MN; (b) the perimeter of $\triangle PAB$; (c) the area of $\triangle PAB$; (d) the area of trapezoid $ABNM$.

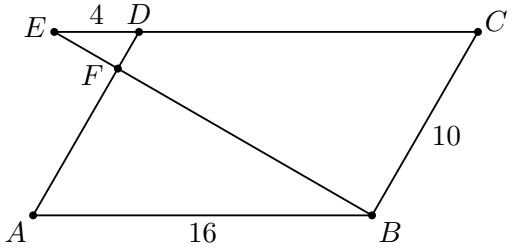


(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Strategy. Check the four quantities one at a time. The midsegment theorem fixes MN, the fixed distance between the two parallel lines fixes the area of $\triangle PAB$, and triangle PMN is a half-size copy of triangle PAB.

Solution. Take the four quantities in turn.
(a) The segment MN joins the midpoints of the sides PA and PB, so $MN = \frac{1}{2}AB$ by the midsegment theorem. This length does not depend on P.
(b) As P moves along the line away from A and B, the length PA grows without bound while AB stays the same, so the perimeter $PA + PB + AB$ changes.
(c) The line on which P moves is parallel to AB, so the height of $\triangle PAB$ to the base AB stays the same. By the theorem on equal areas between parallels, the area of $\triangle PAB$ does not change.
(d) The midsegment theorem also gives $MN \parallel AB$, so $\triangle PAB \sim \triangle PMN$ with ratio $\frac{1}{2}$ by the side-splitting theorem. By the theorem on areas of similar triangles, $[PMN] = \frac{1}{4}[PAB]$, so the area of the trapezoid, $[ABNM] = [PAB] - [PMN] = \frac{3}{4}[PAB]$, does not change either.
Only the perimeter changes.
Answer: (B) 1. □

4. (1990 AHSME, Problem #4) Let $ABCD$ be a parallelogram with $\angle ABC = 120°$, $\overline{AB} = 16$, and $BC = 10$. Extend $\overline{CD}$ through D to E so that $DE = 4$. If $\overline{BE}$ intersects $\overline{AD}$ at F, then FD is closest to



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Strategy. The line AD is parallel to BC, so in triangle ECB the segment DF cuts off a smaller triangle at E. Side splitting compares FD with BC.

Solution. Opposite sides of a parallelogram are equal and parallel, so $DC = AB = 16$ and $AD \parallel BC$. The point E lies on the extension of CD beyond D, so D lies between E and

C, and

$$EC = ED + DC = 4 + 16 = 20.$$

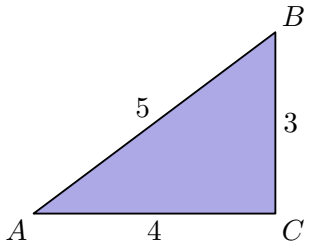
In triangle ECB, the line through D parallel to CB is the line AD, and it meets the side EB at F. By the side-splitting theorem,

$$\frac{FD}{BC} = \frac{ED}{EC} = \frac{4}{20} = \frac{1}{5}, \qquad \text{so} \qquad FD = \frac{10}{5} = 2.$$

So FD is exactly 2. The angle of 120° is not needed.

Answer: (B) 2. □

5. (2018 AMC 10A, Problem #13) A paper triangle with sides of lengths 3, 4, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?



(A) $1 + \frac{1}{2}\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{15}{8}$ **(E)** 2

Strategy. Each point of the crease is equally far from A and B, so the crease lies on the perpendicular bisector of AB. This line forms a right triangle with the leg AC, and that right triangle shares the angle at A with triangle ACB.

Solution. In the figure, $AC = 4$, $CB = 3$, the hypotenuse is $AB = 5$, and the right angle is at C. Folding moves the part of the paper on one side of the crease without stretching it and leaves the crease in place. Since A lands on B, every point X of the crease satisfies $XA = XB$, so the crease lies on the perpendicular bisector of AB. This line passes through the midpoint M of AB, with $AM = \frac{5}{2}$; let it meet line AC at N.

Triangles ACB and AMN have right angles at C and M and share the angle at A, so $\triangle ACB \sim \triangle AMN$ by AA, with $A \to A$, $C \to M$, $B \to N$. The ratio of similarity is $\frac{AM}{AC} = \frac{5/2}{4} = \frac{5}{8}$, so

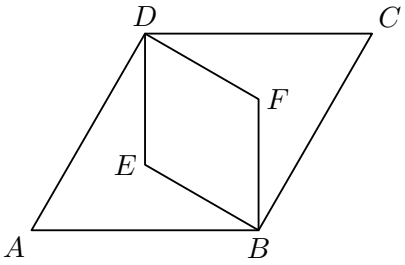
$$AN = \frac{5}{8} \cdot AB = \frac{25}{8} \qquad \text{and} \qquad MN = \frac{5}{8} \cdot CB = \frac{15}{8}.$$

Since $AN = \frac{25}{8} < 4 = AC$, the point N lies on side AC. So the part of the perpendicular bisector inside the triangle is the segment MN, and the crease has length $\frac{15}{8}$.

Answer: (D) $\frac{15}{8}$. □

Intermediate.

6. (2006 AMC 10B, Problem #15) Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 24, and $\angle BAD = 60°$. What is the area of rhombus $BFDE$?



(A) 6 **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** 9 **(E)** $6\sqrt{3}$

Strategy. In similar polygons, corresponding diagonals are in the ratio of similarity. The order of the letters sends A and C to B and D, so the diagonal BD of $BFDE$ corresponds to the diagonal AC of $ABCD$.

Solution. Let s be the side of $ABCD$. Triangle ABD has $AB = AD = s$ and vertex angle $\angle BAD = 60°$, so it is equilateral and $BD = s$. The diagonals of a rhombus bisect each other at right angles, so they meet at the midpoint O of BD, and AO is the altitude of the equilateral triangle ABD from A. By the Pythagorean theorem,

$$AO = \sqrt{s^2 - \frac{s^2}{4}} = \frac{\sqrt{3}}{2}\,s, \qquad \text{so} \qquad AC = 2\,AO = \sqrt{3}\,s.$$

The similarity of $ABCD$ and $BFDE$ matches $A \to B$, $B \to F$, $C \to D$, $D \to E$, so the diagonal AC of $ABCD$ corresponds to the diagonal BD of $BFDE$. By the theorem on areas of similar polygons, corresponding diagonals are in the ratio of similarity k, so

$$k = \frac{BD}{AC} = \frac{s}{\sqrt{3}\,s} = \frac{1}{\sqrt{3}},$$

and the area of $BFDE$ is $k^2 \cdot 24 = \frac{24}{3} = 8$.

Answer: (C) 8. □

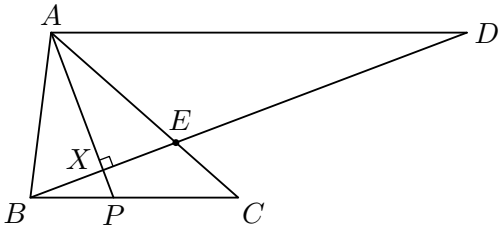
7. (2022 AMC 10A, Problem #13) Let $\triangle ABC$ be a scalene triangle. Point P lies on $\overline{BC}$ so that $\overline{AP}$ bisects $\angle BAC$. The line through B perpendicular to $\overline{AP}$ intersects the line through A parallel to $\overline{BC}$ at point D. Suppose $BP = 2$ and $PC = 3$. What is AD?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

Strategy. Let the line BD meet AC at E. The bisector makes triangle ABE isosceles, so $AE = AB$, and the angle bisector theorem then gives $AE : EC$. The parallel segments AD and CB are joined crosswise by AC and DB, which cross at E.

Solution. By the angle bisector theorem, $AB : AC = BP : PC = 2 : 3$, so $AB < AC$. Also $BC = 2 + 3 = 5$.

Let E be the point of side AC with $AE = AB$. The bisector AP of $\angle BAE$ crosses BE at a point X. Triangles ABX and AEX are congruent by SAS: $AB = AE$, the side AX is common, and the angles at A are equal. So $\angle AXB = \angle AXE$, and since these two angles add up to 180°, each is 90°. Hence BE is the line through B perpendicular to AP, and D lies on line BE, as in the figure.



Since $AE = AB = \frac{2}{3}AC$, the point E divides AC with $AE : EC = 2 : 1$. The segments AD and CB are parallel, and the segments AC and DB join them crosswise and cross at E. By the theorem on the diagonals of a trapezoid,

$$\frac{AD}{CB} = \frac{AE}{EC} = 2, \qquad \text{so} \qquad AD = 2 \cdot 5 = 10.$$

Answer: (C) 10. □

8. (1955 AHSME, Problem #31) An equilateral triangle whose side is 2 is divided into a triangle and a trapezoid by a line drawn parallel to one of its sides. If the area of the trapezoid equals one-half of the area of the original triangle, the length of the median of the trapezoid is:

(A) $\frac{\sqrt{6}}{2}$ **(B)** $\sqrt{2}$ **(C)** $2+\sqrt{2}$ **(D)** $\frac{2+\sqrt{2}}{2}$ **(E)** $\frac{2\sqrt{3}-\sqrt{6}}{2}$

Strategy. The line cuts off a triangle similar to the whole one with half of its area, which gives the ratio of similarity. The median of a trapezoid, its midline, is half the sum of the bases.

Solution. Let the triangle be ABC, and let the line meet AB and AC at D and E, with $DE \parallel BC$. The trapezoid $DBCE$ has half the area of triangle ABC, so triangle ADE has the other half: $[ADE] = \frac{1}{2}[ABC]$. By the side-splitting theorem, $\triangle ABC \sim \triangle ADE$ with some ratio k, and by the theorem on areas of similar triangles, $k^2 = \frac{1}{2}$. So $k = \frac{1}{\sqrt{2}}$ and

$$DE = k \cdot BC = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

The median of trapezoid $DBCE$ joins the midpoints of its legs DB and EC, so it is the midline, and its length is half the sum of the bases:

$$\frac{BC + DE}{2} = \frac{2+\sqrt{2}}{2}.$$

Answer: (D) $\frac{2+\sqrt{2}}{2}$. □

9. (1956 AHSME, Problem #38) In a right triangle with sides a and b, and hypotenuse c, the altitude drawn on the hypotenuse is x. Then:

(A) $ab = x^2$ **(B)** $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$ **(C)** $a^2 + b^2 = 2x^2$ **(D)** $\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{b^2}$

(E) $\frac{1}{x} = \frac{b}{a}$

Strategy. The altitude cuts the hypotenuse into two segments. The altitude-to-the-hypotenuse theorem expresses x^2, a^2 and b^2 through these segments and c; eliminate the segments.

Solution. The altitude cuts the hypotenuse into a segment r next to the leg a and a segment s next to the leg b. By the altitude-to-the-hypotenuse theorem,

$$x^2 = rs, \qquad a^2 = rc, \qquad b^2 = sc.$$

Multiplying the last two equations gives $a^2b^2 = rs \cdot c^2 = x^2c^2$, so $ab = xc$. (Both ab and xc are twice the area of the triangle, which confirms this.) By the Pythagorean theorem, $c^2 = a^2 + b^2$, so

$$\frac{1}{x^2} = \frac{c^2}{a^2b^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2},$$

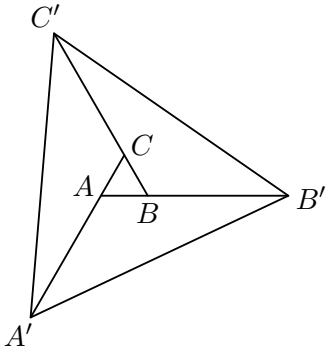
which is (D). The other four relations fail for the right triangle with $a = 3$, $b = 4$ and $c = 5$, where $x = \frac{12}{5}$ and $\frac{1}{x} = \frac{5}{12}$: in (A), $ab = 12$ but $x^2 = \frac{144}{25}$; in (B), $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$; in (C), $a^2 + b^2 = 25$ but $2x^2 = \frac{288}{25}$; and in (E), $\frac{b}{a} = \frac{4}{3}$.

Answer: (D) $\dfrac{1}{x^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$. □

10. (2017 AMC 10B, Problem #19) Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3 \cdot AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3 \cdot BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3 \cdot CA$.
What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?
(A) 9 : 1 **(B)** 16 : 1 **(C)** 25 : 1 **(D)** 36 : 1 **(E)** 37 : 1

Strategy. The segments AB', BC' and CA' cut triangle $A'B'C'$ into triangle ABC and three corner triangles. Find the area of a corner triangle in two steps: equal heights first, then the common-angle theorem.

Solution. Let $K = [ABC]$ and $s = AB = BC = CA$. The points B, C and A lie on the segments AB', BC' and CA', and these three segments cut triangle $A'B'C'$ into triangle ABC and the three triangles $BB'C'$, $CC'A'$ and $AA'B'$, as in the figure.



Consider the corner triangle $BB'C'$. Triangles $B'BC$ and ABC have the common vertex C and their bases $BB' = 3s$ and $BA = s$ on line AB, so $[B'BC] = 3K$ by the equal-heights theorem. Triangles $BB'C'$ and $BB'C$ share the angle at B, because C lies on ray BC', and $BC' = BC + CC' = 4s$. By the common-angle theorem,

$$\frac{[BB'C']}{[BB'C]} = \frac{BB'}{BB'} \cdot \frac{BC'}{BC} = 4, \qquad \text{so} \qquad [BB'C'] = 12K.$$

The same argument gives $[CC'A'] = [AA'B'] = 12K$. Therefore

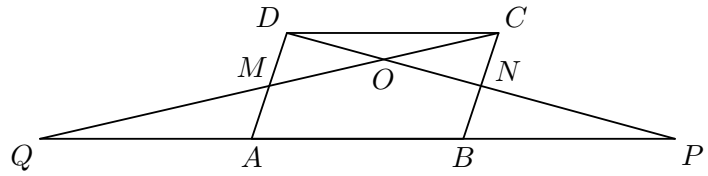
$$[A'B'C'] = K + 3 \cdot 12K = 37K,$$

and the ratio is $37 : 1$.

Answer: (E) $37 : 1$. □

Advanced.

11. (1974 AHSME, Problem #25) In parallelogram $ABCD$ of the accompanying diagram, line DP is drawn bisecting BC at N and meeting AB (extended) at P. From vertex C, line CQ is drawn bisecting side AD at M and meeting AB (extended) at Q. Lines DP and CQ meet at O. If the area of parallelogram $ABCD$ is k, then the area of triangle QPO is equal to



(A) k **(B)** $\frac{6k}{5}$ **(C)** $\frac{9k}{8}$ **(D)** $\frac{5k}{4}$ **(E)** $2k$

Strategy. The midpoints give two pairs of congruent triangles, which show that $QP = 3\,DC$. The segments DC and QP are parallel and are joined crosswise by DP and CQ, so the theorem on the diagonals of a trapezoid locates O between them.

Solution. Let $b = AB = DC$, and let h be the distance between the parallel lines AB and DC, so that $k = bh$.

Triangles NBP and NCD have $NB = NC$, equal vertical angles at N, and equal alternate interior angles $\angle NBP = \angle NCD$, since $BP \parallel CD$. So they are congruent by ASA, and $BP = CD = b$. In the same way, triangles MAQ and MDC are congruent, and $AQ = DC = b$. Hence

$$QP = QA + AB + BP = 3b.$$

The segments QP and DC are parallel, and they are joined crosswise by QC and PD, which meet at O. As in the proof of the theorem on the diagonals of a trapezoid, the alternate interior angles give $\triangle OCD \sim \triangle OQP$ by AA, with ratio $\frac{QP}{CD} = 3$. By the theorem on areas of similar triangles, the altitude of $\triangle OQP$ from O is 3 times the altitude of $\triangle OCD$ from O. These two altitudes lie on one line perpendicular to AB and DC, on opposite sides of O, and together they span the distance h. So the altitude of $\triangle OQP$ from O is $\frac{3}{4}h$, and

$$[QPO] = \frac{1}{2} \cdot 3b \cdot \frac{3}{4}h = \frac{9}{8}\,bh = \frac{9k}{8}.$$

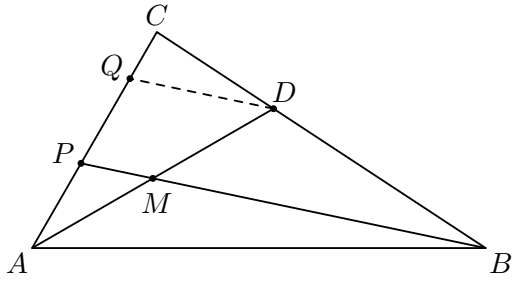
Answer: (C) $\frac{9k}{8}$. □

12. (2011 AIME II, Problem #4) In triangle ABC, $AB = \frac{20}{11}AC$. The angle bisector of angle A intersects $\overline{BC}$ at point D, and point M is the midpoint of $\overline{AD}$. Let P be the point of intersection of $\overline{AC}$ and line BM. The ratio of CP to PA can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Strategy. The angle bisector theorem gives $BD : DC$. Let the line through D parallel to BP meet AC at Q: in triangle ADQ the line BM then makes P the midpoint of AQ, and in triangle CBP the line DQ splits CP in the ratio $CD : DB$.

Solution. By the angle bisector theorem, $BD : DC = AB : AC = 20 : 11$.

Let the line through D parallel to BP meet AC at Q, as in the figure.



In triangle ADQ, the point M is the midpoint of the side AD, and the line BM through M is parallel to DQ. By the converse part of the midsegment theorem, this line meets AQ at its midpoint, so P is the midpoint of AQ and $AP = PQ$.

In triangle CBP, the points D and Q lie on the sides CB and CP, and $DQ \parallel BP$. By the side-splitting theorem,

$$\frac{CQ}{QP} = \frac{CD}{DB} = \frac{11}{20}.$$

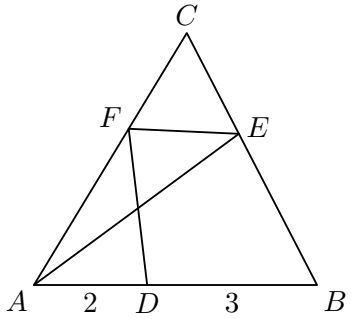
Let $PQ = 20t$. Then $CQ = 11t$ and $PA = PQ = 20t$, so

$$\frac{CP}{PA} = \frac{CQ + QP}{PA} = \frac{31t}{20t} = \frac{31}{20}.$$

Since 31 and 20 are relatively prime, $m + n = 31 + 20 = 51$.

Answer: 51. □

13. (1983 AHSME, Problem #28) Triangle ABC in the figure has area 10. Points D, E and F, all distinct from A, B and C, are on sides AB, BC and CA respectively, and $AD = 2$, $DB = 3$. If triangle ABE and quadrilateral $DBEF$ have equal areas, then that area is



(A) 4 **(B)** 5 **(C)** 6 **(D)** $\frac{5}{3}\sqrt{10}$ **(E)** not uniquely determined

Strategy. Remove triangle DBE from both of the equal areas. What is left shows that A and F are equally far from line DE, so $DE \parallel AC$, and side splitting then gives $BE : BC$.

Solution. Since D lies between A and B, and E lies between B and C, the line DE has B on one side and A and C on the other. The whole side CA lies on the side of A and C, so F lies on the same side of line DE as A, opposite to B.

So the segment DE cuts triangle ABE into triangles ADE and DBE, and it cuts quadrilateral $DBEF$ into triangles DBE and DEF. Hence

$$[ADE] + [DBE] = [ABE] = [DBEF] = [DBE] + [DEF],$$

and $[ADE] = [FDE]$. These two triangles have the common base DE, and A and F lie on the same side of line DE, so $AF \parallel DE$ by the theorem on equal areas between parallels. The line AF is the line AC, so $DE \parallel AC$.

In triangle BAC, the points D and E lie on the sides BA and BC, and $DE \parallel AC$. By the side-splitting theorem,

$$\frac{BE}{BC} = \frac{BD}{BA} = \frac{3}{5}.$$

Triangles ABE and ABC have the common vertex A and their bases BE and BC on line BC, so by the equal-heights theorem

$$[ABE] = \frac{3}{5}\,[ABC] = \frac{3}{5} \cdot 10 = 6.$$

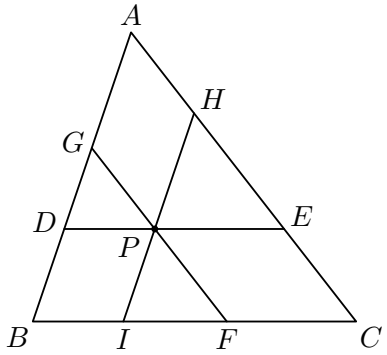
Conversely, when $DE \parallel AC$, the theorem on equal areas between parallels gives $[ADE] = [FDE]$ for every position of F, so the two areas are equal and the value 6 does occur.

Answer: (C) 6. □

14. (1986 AIME, Problem #9) In $\triangle ABC$, $AB = 425$, $BC = 450$, and $AC = 510$. An interior point P is drawn, and segments are drawn through P parallel to the sides of the triangle. If these three segments have equal length d, find d.

Strategy. The three lines through P cut off three small triangles similar to ABC, and their ratios of similarity add up to 1. Each segment through P consists of sides of two of the small triangles, so its length is a side of ABC times the sum of two of the ratios.

Solution. Name the ends of the three segments as in the figure: the segment parallel to BC is DE, with D on AB and E on CA; the segment parallel to CA is GF, with G on AB and F on BC; and the segment parallel to AB is IH, with I on BC and H on CA.



The small triangles PDG, PEH and PIF have their sides parallel to the sides of ABC, so their angles equal those of ABC (corresponding angles), and each is similar to ABC by AA. Let k_1, k_2 and k_3 be their ratios of similarity to ABC.

The quadrilaterals $BIPD$ and $FCEP$ have both pairs of opposite sides parallel, so they are parallelograms, and $BI = DP$ and $FC = PE$. The sides of the small triangles parallel to BC are $DP = k_1\,BC$, $PE = k_2\,BC$ and $IF = k_3\,BC$. Hence

$$BC = BI + IF + FC = DP + IF + PE = (k_1 + k_2 + k_3)\,BC,$$

so $k_1 + k_2 + k_3 = 1$. Each of the three segments through P is made of two sides of small triangles. The sides of PDG and PIF parallel to CA are GP and PF, and the sides of PIF and PEH parallel to AB are IP and PH, so

$$DE = (k_1 + k_2)\,BC, \qquad GF = (k_1 + k_3)\,CA, \qquad IH = (k_3 + k_2)\,AB.$$

All three are equal to d, and adding the three equations $\frac{d}{BC} = k_1 + k_2$, $\frac{d}{CA} = k_1 + k_3$ and $\frac{d}{AB} = k_2 + k_3$ gives

$$\frac{d}{BC} + \frac{d}{CA} + \frac{d}{AB} = 2(k_1 + k_2 + k_3) = 2.$$

Here

$$\frac{1}{450} + \frac{1}{510} + \frac{1}{425} = \frac{17 + 15 + 18}{7650} = \frac{50}{7650} = \frac{1}{153},$$

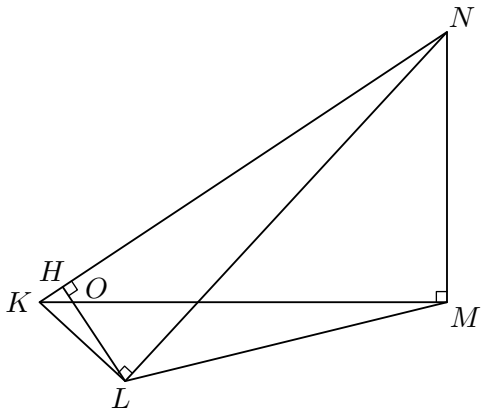
so $\frac{d}{153} = 2$ and $d = 306$. As a check, the three equations then give $k_1 = \frac{7}{25}$, $k_2 = \frac{2}{5}$ and $k_3 = \frac{8}{25}$, all positive, so such a point P exists.

Answer: 306. □

15. (2019 AIME I, Problem #6) In convex quadrilateral $KLMN$, side $\overline{MN}$ is perpendicular to diagonal $\overline{KM}$, side $\overline{KL}$ is perpendicular to diagonal $\overline{LN}$, $MN = 65$, and $KL = 28$. The line through L perpendicular to side $\overline{KN}$ intersects diagonal $\overline{KM}$ at O with $KO = 8$. Find MO.

Strategy. Let H be the foot of the perpendicular from L to KN. Then LH is the altitude to the hypotenuse of the right triangle KLN, so $KL^2 = KH \cdot KN$. A second pair of right triangles, with the angle at K in common, shows that $KH \cdot KN = KO \cdot KM$.

Solution. Let H be the point where the perpendicular from L meets KN. Then O lies on line LH, as in the figure.



Triangle KLN has a right angle at L, so H lies on its hypotenuse KN, and by the altitude-to-the-hypotenuse theorem

$$KL^2 = KH \cdot KN.$$

Triangles KHO and KMN have right angles at H and M, and they share the angle at K, since H lies on ray KN and O lies on ray KM. So $\triangle KHO \sim \triangle KMN$ by AA, with $K \to K$, $H \to M$, $O \to N$, and

$$\frac{KH}{KM} = \frac{KO}{KN}, \qquad \text{so} \qquad KH \cdot KN = KO \cdot KM.$$

The two equations give $KO \cdot KM = KL^2$, that is, $8 \cdot KM = 28^2 = 784$, so $KM = 98$. Since O lies on the diagonal KM,

$$MO = KM - KO = 98 - 8 = 90.$$

The length $MN = 65$ is not needed.

Answer: 90. □

2. Answer Key

Problem	Letter	Value
1	D	$6\frac{2}{3}$
2	D	33.3
3	B	1
4	B	2
5	D	$\frac{15}{8}$
6	C	8
7	C	10
8	D	$\frac{2+\sqrt{2}}{2}$
9	D	$\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{b^2}$
10	E	$37:1$
11	C	$\frac{9k}{8}$
12		51
13	C	6
14		306
15		90