TRIANGLE CENTERS, MEDIANS, AND CEVIANS FOR AMC 10: PRACTICE SET

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1. Problems

Beginner.

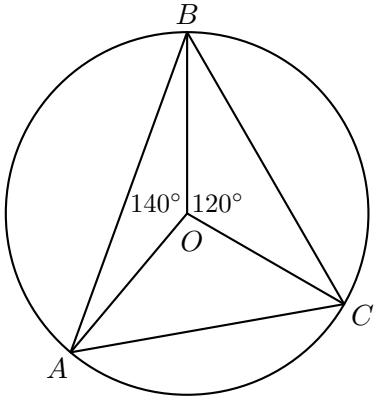
1. (1966 AHSME, Problem #14) The length of rectangle $ABCD$ is 5 inches and its width is 3 inches. Diagonal AC is divided into three equal segments by points E and F. The area of triangle BEF, expressed in square inches, is:

(A) $\frac{3}{2}$ **(B)** $\frac{5}{3}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{1}{3}\sqrt{34}$ **(E)** $\frac{1}{3}\sqrt{68}$

2. (1966 AHSME, Problem #11) The sides of triangle BAC are in the ratio $2:3:4$. BD is the angle-bisector drawn to the shortest side AC, dividing it into segments AD and CD. If the length of AC is 10, then the length of the longer segment of AC is:

(A) $3\frac{1}{2}$ **(B)** 5 **(C)** $5\frac{5}{7}$ **(D)** 6 **(E)** $7\frac{1}{2}$

3. (2007 AMC 10B, Problem #4) The point O is the center of the circle circumscribed about $\triangle ABC$, with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ$, as shown. What is the degree measure of $\angle ABC$?



(A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 60

4. (1964 AHSME, Problem #13) A circle is inscribed in a triangle with side lengths 8, 13, and 17. Let the segments of the side of length 8, made by a point of tangency, be r and s, with $r < s$. What is the ratio $r : s$?

(A) $1:3$ **(B)** $2:5$ **(C)** $1:2$ **(D)** $2:3$ **(E)** $3:4$

5. (1961 AHSME, Problem #21) Medians AD and CE of triangle ABC intersect in M. The midpoint of AE is N. Let the area of triangle MNE be k times the area of triangle ABC. Then k equals:

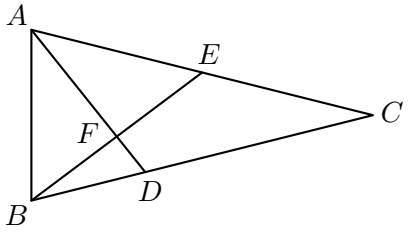
(A) $\frac{1}{6}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{16}$

Intermediate.

6. (2024 AMC 10B, Problem #10) Quadrilateral $ABCD$ is a parallelogram, and E is the midpoint of the side $\overline{AD}$. Let F be the intersection of lines EB and AC. What is the ratio of the area of quadrilateral $CDEF$ to the area of triangle CFB?

(A) $5:4$ **(B)** $4:3$ **(C)** $3:2$ **(D)** $5:3$ **(E)** $2:1$

7. (1980 AHSME, Problem #21) In triangle ABC, $\angle CBA = 72°$, E is the midpoint of side AC, and D is a point on side BC such that $2BD = DC$; AD and BE intersect at F. The ratio of the area of $\triangle BDF$ to the area of quadrilateral $FDCE$ is



(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{5}$ **(E)** none of these

8. (2018 AMC 10A, Problem #16) Right triangle ABC has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$?

(A) 5 **(B)** 8 **(C)** 12 **(D)** 13 **(E)** 15

9. (2003 AMC 10A, Problem #17) The number of inches in the perimeter of an equilateral triangle equals the number of square inches in the area of its circumscribed circle. What is the radius, in inches, of the circle?

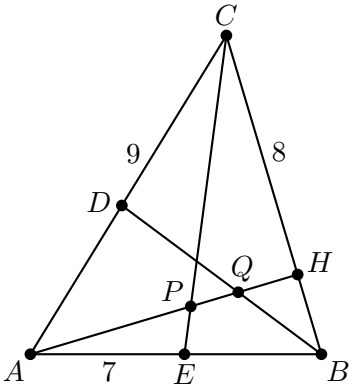
(A) $\frac{3\sqrt{2}}{\pi}$ **(B)** $\frac{3\sqrt{3}}{\pi}$ **(C)** $\sqrt{3}$ **(D)** $\frac{6}{\pi}$ **(E)** $\sqrt{3}\pi$

10. (2024 AMC 12B, Problem #20) Suppose A, B, and C are points in the plane with $AB = 40$ and $AC = 42$, and let x be the length of the line segment from A to the midpoint of $\overline{BC}$. Define a function f by letting $f(x)$ be the area of $\triangle ABC$. Then the domain of f is an open interval (p, q), and the maximum value r of $f(x)$ occurs at $x = s$. What is $p + q + r + s$?

(A) 909 **(B)** 910 **(C)** 911 **(D)** 912 **(E)** 913

Advanced.

11. (2016 AMC 12B, Problem #17) In $\triangle ABC$ shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. What is PQ?

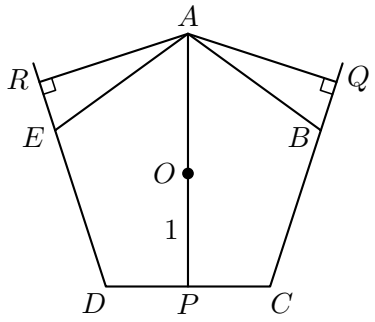


(A) 1 **(B)** $\frac{5}{8}\sqrt{3}$ **(C)** $\frac{4}{5}\sqrt{2}$ **(D)** $\frac{8}{15}\sqrt{5}$ **(E)** $\frac{6}{5}$

12. (1964 AHSME, Problem #35) The sides of a triangle are of lengths 13, 14, and 15. The altitudes of the triangle meet at point H. If AD is the altitude to the side of length 14, what is the ratio $HD : HA$?

(A) $3 : 11$ **(B)** $5 : 11$ **(C)** $1 : 2$ **(D)** $2 : 3$ **(E)** $25 : 33$

13. (1986 AHSME, Problem #28) $ABCDE$ is a regular pentagon. AP, AQ and AR are the perpendiculars dropped from A onto CD, CB extended and DE extended, respectively. Let O be the center of the pentagon. If $OP = 1$, then $AO + AQ + AR$ equals

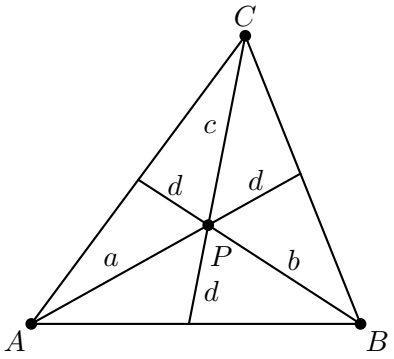


(A) 3 **(B)** $1 + \sqrt{5}$ **(C)** 4 **(D)** $2 + \sqrt{5}$ **(E)** 5

14. (2022 AMC 10B, Problem #20) Let $ABCD$ be a rhombus with $\angle ADC = 46°$. Let E be the midpoint of $\overline{CD}$, and let F be the point on $\overline{BE}$ such that $\overline{AF}$ is perpendicular to $\overline{BE}$. What is the degree measure of $\angle BFC$?

(A) 110 **(B)** 111 **(C)** 112 **(D)** 113 **(E)** 114

15. (1988 AIME, Problem #12) Let P be an interior point of triangle ABC and extend lines from the vertices through P to the opposite sides. Let a, b, c, and d denote the lengths of the segments indicated in the figure. Find the product abc if $a + b + c = 43$ and $d = 3$.



2. Answer Key

Problem	Letter	Value
1	C	$\frac{5}{2}$
2	C	$5\frac{5}{7}$
3	D	50
4	A	$1:3$
5	D	$\frac{1}{12}$
6	A	$5:4$
7	A	$\frac{1}{5}$
8	D	13
9	B	$\frac{3\sqrt{3}}{\pi}$
10	C	911
11	D	$\frac{8}{15}\sqrt{5}$
12	B	$5:11$
13	C	4
14	D	113
15		441