STARS AND BARS, DISTRIBUTIONS, AND CIRCULAR ARRANGEMENTS FOR AMC 10: PRACTICE SET

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1. Problems

Beginner.

1. (2004 AMC 10B, Problem #2) How many two-digit positive integers have at least one 7 as a digit?

(A) 10 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 30

2. Identical balls are stacked in the shape of a triangular pyramid. The top layer is a single ball, and each layer below it is a triangle with one more ball on each side than the layer above it, so the layers contain 1, 3, 6, 10, ... balls. The bottom layer has 20 balls along each side. How many balls are in the pyramid?

3. Five married couples are to be seated at a round table with ten chairs so that each person sits next to his or her spouse. Two seatings are considered the same if one is a rotation of the other. How many different seatings are possible?

4. (2017 AMC 10B, Problem #13) There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three.
There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5. Ten lockers stand in a row. Some of them, possibly none, are to be painted red so that no two neighboring lockers are both red. In how many ways can the set of red lockers be chosen?

Intermediate.

6. (2023 AMC 10B, Problem #11) Suzanne went to the bank and withdrew $800. The teller gave her this amount using $20 bills, $50 bills, and $100 bills, with at least one of each denomination. How many different collections of bills could Suzanne have received?

(A) 45 **(B)** 21 **(C)** 36 **(D)** 28 **(E)** 32

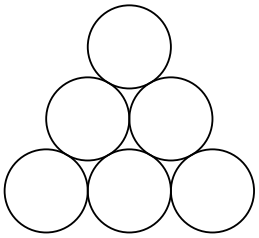
7. Four people are to be seated at a round table with 12 equally spaced chairs so that no two of them sit in neighboring chairs. Two seatings are considered the same if one is a rotation of the other. How many different seatings are possible?

8. (2010 AMC 10B, Problem #22) Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?

(A) 1930 **(B)** 1931 **(C)** 1932 **(D)** 1933 **(E)** 1934

9. (1990 AIME, Problem #8) In a shooting match, eight clay targets are arranged in two hanging columns of three targets each and one column of two targets. A marksman is to break all the targets according to the following rules:
(1) The marksman first chooses a column from which a target is to be broken.
(2) The marksman must then break the lowest remaining target in the chosen column.
If the rules are followed, in how many different orders can the eight targets be broken?

10. (2017 AMC 10B, Problem #18) In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?



(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

Advanced.

11. (2026 AIME I, Problem #2) Find the number of positive integer palindromes written in base 10, with no zero digits, and whose digits add up to 13. For example, 42124 has these properties. Recall that a palindrome is a number whose representation reads the same from left to right as from right to left.

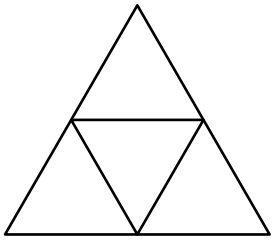
12. (2024 AMC 10B, Problem #22) A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

13. (2025 AIME I, Problem #3) The 9 members of a baseball team went to an ice-cream parlor after their game. Each player had a single scoop cone of chocolate, vanilla, or strawberry ice cream. At least one player chose each flavor, and the number of players who chose chocolate was greater than the number of players who chose vanilla, which was greater than the number of players who chose strawberry. Let N be the number of different assignments of flavors to players that meet these conditions. Find the remainder when N is divided by 1000.

14. (1996 AIME, Problem #7) Two of the squares of a 7×7 checkerboard are painted yellow, and the rest are painted green. Two color schemes are equivalent if one can be obtained from the other by applying a rotation in the plane of the board. How many inequivalent color schemes are possible?

15. (2006 AIME II, Problem #8) There is an unlimited supply of congruent equilateral triangles made of colored paper. Each triangle is a solid color with the same color on both sides of the paper. A large equilateral triangle is constructed from four of these paper triangles as shown. Two large triangles are considered distinguishable if it is not possible to place one on the other, using translations, rotations, and/or reflections, so that their corresponding small triangles are of the same color. Given that there are six different colors of triangles from which to choose, how many distinguishable large equilateral triangles can be constructed?



2. Answer Key

Problem	Letter	Value
1	B	18
2		1540
3		768
4	C	3
5		144
6	B	21
7		210
8	C	1932
9		560
10	D	12
11		62
12	A	5
13		16
14		300
15		336