COMBINATIONS AND THE BINOMIAL THEOREM FOR AMC 10: PRACTICE SET

ANDY JO

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1. Problems

Beginner.

1. (2013 AMC 10A, Problem #7) A student must choose a program of four courses from a menu of courses consisting of English, Algebra, Geometry, History, Art, and Latin. This program must contain English and at least one mathematics course. In how many ways can this program be chosen?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 16

2. (1950 AHSME, Problem #45) The number of diagonals that can be drawn in a polygon of 100 sides is:

(A) 4850 **(B)** 4950 **(C)** 9900 **(D)** 98 **(E)** 8800

3. (2007 AMC 10B, Problem #8) On the trip home from the meeting where this AMC 10 was constructed, the Contest Chair noted that his airport parking receipt had digits of the form $bbcac$, where $0 \leq a < b < c \leq 9$, and b was the average of a and c. How many different five-digit numbers satisfy all these properties?

(A) 12 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

4. (1961 AHSME, Problem #7) When simplified, the third term in the expansion of

$$\left(\frac{a}{\sqrt{x}} - \frac{\sqrt{x}}{a^2}\right)^6$$

is:

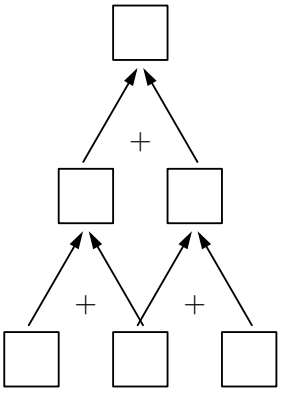
(A) $\frac{15}{x}$ **(B)** $-\frac{15}{x}$ **(C)** $-\frac{6x^2}{a^9}$ **(D)** $\frac{20}{a^3}$ **(E)** $-\frac{20}{a^3}$

5. (2005 AMC 10B, Problem #18) All of David's telephone numbers have the form 555–abc–$defg$, where a, b, c, d, e, f, and g are distinct digits and in increasing order, and none is either 0 or 1. How many different telephone numbers can David have?

(A) 1 **(B)** 2 **(C)** 7 **(D)** 8 **(E)** 9

Intermediate.

6. (2006 AMC 8, Problem #22) Three different one-digit positive integers are placed in the bottom row of cells. Numbers in adjacent cells are added and the sum is placed in the cell above them. In the second row, continue the same process to obtain a number in the top cell. What is the difference between the largest and smallest numbers possible in the top cell?



(A) 16 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 35

7. (1975 AHSME, Problem #5) The polynomial $(x+y)^9$ is expanded in decreasing powers of x. The second and third terms have equal values when evaluated at $x=p$ and $y=q$, where p and q are positive numbers whose sum is one. What is the value of p?
(A) $\frac{1}{5}$ **(B)** $\frac{4}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{8}{9}$

8. (1994 AHSME, Problem #15) For how many n in $\{1,2,3,\ldots,100\}$ is the tens digit of n^2 odd?
(A) 10 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 50

9. (1989 AIME, Problem #2) Ten points are marked on a circle. How many distinct convex polygons of three or more sides can be drawn using some (or all) of the ten points as vertices?

10. (2005 AIME II, Problem #1) A game uses a deck of n different cards, where n is an integer and $n \geq 6$. The number of possible sets of 6 cards that can be drawn from the deck is 6 times the number of possible sets of 3 cards that can be drawn. Find n.

Advanced.

11. (2003 AMC 12A, Problem #20) How many 15-letter arrangements of 5 A's, 5 B's, and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters?
(A) $\sum_{k=0}^{5}\binom{5}{k}^3$ **(B)** $3^5\cdot 2^5$ **(C)** 2^{15} **(D)** $\frac{15!}{(5!)^3}$ **(E)** 3^{15}

12. (1976 AHSME, Problem #23) For integers k and n such that $1 \leq k < n$, let

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Then $\left(\frac{n-2k-1}{k+1}\right)\binom{n}{k}$ is an integer
(A) for all k and n **(B)** for all even values of k and n, but not for all k and n
(C) for all odd values of k and n, but not for all k and n
(D) if $k=1$ or $n-1$, but not for all odd values of k and n
(E) if n is divisible by k, but not for all even values of k and n

13. (2006 AIME II, Problem #4) Let $(a_1,a_2,a_3,\ldots,a_{12})$ be a permutation of $(1,2,3,\ldots,12)$ for which

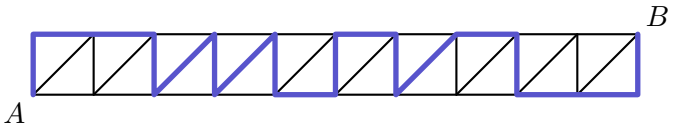
$$a_1 > a_2 > a_3 > a_4 > a_5 > a_6$$

and

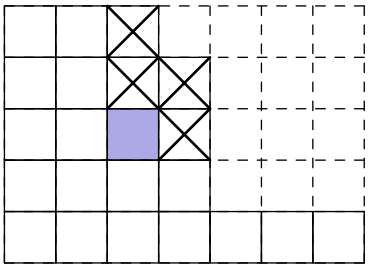
$$a_6 < a_7 < a_8 < a_9 < a_{10} < a_{11} < a_{12}.$$

An example of such a permutation is $(6,5,4,3,2,1,7,8,9,10,11,12)$. Find the number of such permutations.

14. (2026 AIME II, Problem #2) The figure below shows a grid of 10 squares in a row. Each square has a diagonal connecting its lower left vertex to its upper right vertex. A bug moves along the line segments from vertex to vertex, never traversing the same segment twice and never moving from right to left along a horizontal or diagonal segment. Let N be the number of paths the bug can take from the lower left corner (A) to the upper right corner (B). One such path from A to B is shown by the thick line segments in the figure. Find $\sqrt{N}$.



15. (1992 AIME, Problem #12) In a game of Chomp, two players alternately take bites from a 5-by-7 grid of unit squares. To take a bite, a player chooses one of the remaining squares, then removes ("eats") all squares in the quadrant defined by the left edge (extended upward) and the lower edge (extended rightward) of the chosen square. For example, the bite determined by the shaded square in the diagram would remove the shaded square and the four squares marked by $\times$. (The squares with two or more dotted edges have been removed from the original board in previous moves.)



The object of the game is to make one's opponent take the last bite. The diagram shows one of the many subsets of the set of 35 unit squares that can occur during the game of Chomp. How many different subsets are there in all? Include the full board and empty board in your count.

2. Answer Key

Problem	Letter	Value
1	C	9
2	A	4850
3	D	20
4	A	$15/x$
5	D	8
6	D	26
7	B	$4/5$
8	B	20
9		968
10		13
11	A	$\sum_{k=0}^{5} \binom{5}{k}^3$
12	A	for all k and n
13		462
14		243
15		792