

COUNTING PRINCIPLES AND PERMUTATIONS FOR AMC 10

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1. THE ADDITION AND MULTIPLICATION PRINCIPLES

Most counting arguments are built from two rules. The Addition Principle handles a count that splits into separate cases, and the Multiplication Principle handles a count in which each object is made by a sequence of choices.

1.1. The Addition Principle.

Theorem 1.1.1 (Addition Principle). *Suppose the objects being counted are divided into groups so that every object belongs to exactly one group. Then the number of objects is the sum of the numbers of objects in the groups.*

For example, a student who chooses one book from a shelf of 4 novels and 3 biographies has $4 + 3 = 7$ choices.

This principle is the basis of **casework**: split the objects into cases, count each case, and add the counts. The cases must not overlap, and together they must include every object. If two cases overlap, the objects they share are counted twice.

1.2. The Multiplication Principle.

Theorem 1.2.1 (Multiplication Principle). *Suppose each object being counted is made by a sequence of k choices, and different sequences of choices make different objects. If the first choice has m_1 options and, whatever the earlier choices were, the second choice has m_2 options, the third has m_3 , and so on up to m_k options for the last choice, then the number of objects is*

$$m_1 \cdot m_2 \cdots m_k.$$

For example, a code of three different letters can be made in $26 \cdot 25 \cdot 24 = 15,600$ ways: there are 26 choices for the first letter, 25 for the second (any letter except the first), and 24 for the third.

In this example the letters available for the second position depend on the first letter, but their number is always 25. The options for a choice may depend on the earlier choices; only the number of options must stay the same. If the number of options changes with the earlier choices, split the count into cases with the Addition Principle, or make the choices in a different order.

Many problems use both principles: the Addition Principle splits the count into cases, and the Multiplication Principle counts each case as a sequence of choices.

Example.

Problem 1.2.2 (2022 AMC 10B, Problem #3). How many three-digit positive integers have an odd number of even digits?

(A) 150 (B) 250 (C) 350 (D) 450 (E) 550

Strategy. Among three digits, an odd number of even digits means exactly one even digit or exactly three. Count these two cases with the Multiplication Principle and add.

Solution. The hundreds digit is one of $1, 2, \dots, 9$, so it has 4 even options (2, 4, 6, 8) and 5 odd options. The tens and units digits are each one of $0, 1, \dots, 9$, with 5 even and 5 odd options.

Case 1: three even digits. There are $4 \cdot 5 \cdot 5 = 100$ such integers.

Case 2: exactly one even digit. If the even digit is the hundreds digit, there are $4 \cdot 5 \cdot 5 = 100$ integers. If it is the tens digit, the hundreds and units digits are odd, so there are $5 \cdot 5 \cdot 5 = 125$ integers, and if it is the units digit, there are also 125. This case gives $100 + 125 + 125 = 350$ integers.

The two cases do not overlap, so by the Addition Principle there are $100 + 350 = 450$ integers.

There is also a shorter count. Choose the hundreds and tens digits first, in $9 \cdot 10 = 90$ ways. Exactly 5 of the 10 units digits then make the number of even digits odd: the even ones if the

first two digits contain an even number of even digits, and the odd ones otherwise. This gives $90 \cdot 5 = 450$ again.

Answer: (D) 450. □

2. FACTORIALS AND PERMUTATIONS

2.1. Factorials.

Definition 2.1.1. For a positive integer n , the **factorial** $n!$ (read “ n factorial”) is the product of the integers from 1 to n :

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdots 2 \cdot 1.$$

In addition, $0! = 1$ by definition.

The table lists the first eleven values.

n	0	1	2	3	4	5	6	7	8	9	10
$n!$	1	1	2	6	24	120	720	5,040	40,320	362,880	3,628,800

Each factorial is a multiple of the one before it: $n! = n \cdot (n - 1)!$. Setting $n = 1$ gives $1! = 1 \cdot 0!$, which is why $0!$ is defined to be 1. The same relation simplifies quotients of factorials, for example

$$\frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720.$$

2.2. Arrangements of Distinct Objects.

Definition 2.2.1. A **permutation** of a collection of objects is an arrangement of the objects in a row. Two permutations are different when some position holds different objects.

Theorem 2.2.2. *There are $n!$ permutations of n distinct objects.*

Proof. Fill the positions from left to right. The first position can hold any of the n objects, the second any of the $n - 1$ objects that remain, and so on, until 1 object is left for the last position. By the Multiplication Principle there are $n(n - 1) \cdots 2 \cdot 1 = n!$ arrangements. □

The same argument, stopped after k positions, counts arrangements of only some of the objects.

Theorem 2.2.3. *The number of ways to choose k of n distinct objects and arrange them in a row is*

$$P(n, k) = n(n - 1)(n - 2) \cdots (n - k + 1) = \frac{n!}{(n - k)!}.$$

The product has k factors, one for each position. For example, a club of 10 members can choose a president, a vice president and a secretary, three different people, in $P(10, 3) = 10 \cdot 9 \cdot 8 = 720$ ways, since the three offices are three positions to fill in order.

When all the permutations of some letters are listed in alphabetical order, the position of one of them in the list can be found by counting the permutations before it in blocks, one block for each choice of the first letters.

Example.

Problem 2.2.4 (2002 AMC 10B, Problem #9). Using the letters A , M , O , S , and U , we can form 120 five-letter “words.” If these “words” are arranged in alphabetical order, then the “word” $USAMO$ occupies which position?

(A) 112 (B) 113 (C) 114 (D) 115 (E) 116

Strategy. Count the words that come before *USAMO*. Group them by their first letter, and group the words that begin with *U* by their second letter.

Solution. In alphabetical order the letters are *A, M, O, S, U*. The $5! = 120$ words are ordered first by their first letter, then by their second letter, and so on.

Every word that begins with *A, M, O* or *S* comes before *USAMO*. For each of these 4 first letters, the other four letters can be arranged in $4! = 24$ ways, so there are $4 \cdot 24 = 96$ such words.

Among the words that begin with *U*, those whose second letter is *A, M* or *O* come before *USAMO*, whose second letter is *S*. For each of these 3 second letters, the remaining three letters can be arranged in $3! = 6$ ways, which gives $3 \cdot 6 = 18$ words.

In *USAMO* the last three letters *A, M, O* are in alphabetical order, so *USAMO* is the first of the words that begin with *US*. Hence exactly $96 + 18 = 114$ words come before *USAMO*, and it is in position 115.

Answer: (D) 115. □

3. ARRANGEMENTS WITH RESTRICTIONS

Many problems put conditions on an arrangement: an object must go in a certain place, some objects must be next to each other, or some objects must not be next to each other. Each kind of condition has a standard method.

3.1. Restricted Positions. When some objects or positions have restrictions, make the restricted choices first and the unrestricted choices afterward. The unrestricted choices then have a fixed number of options, so the Multiplication Principle applies. If the restricted choices are left for last, the number of options for them can depend on the earlier choices.

For example, suppose five people stand in a line and Ann must stand at one of the two ends. Place Ann first, in 2 ways, and then arrange the other four people in the remaining four places, in $4! = 24$ ways. There are $2 \cdot 24 = 48$ lines.

If several restricted choices affect one another, use casework on them and count each case in the same way.

Examples.

Problem 3.1.1 (2006 AMC 12B, Problem #7). Mr. and Mrs. Lopez have two children. When they get into their family car, two people sit in the front, and the other two sit in the back. Either Mr. Lopez or Mrs. Lopez must sit in the driver's seat. How many seating arrangements are possible?

(A) 4 (B) 12 (C) 16 (D) 24 (E) 48

Strategy. Only the driver's seat is restricted, so fill it first.

Solution. The driver's seat must be taken by Mr. Lopez or Mrs. Lopez, so it can be filled in 2 ways. The other three people can then sit in the remaining three seats in $3! = 6$ ways. By the Multiplication Principle there are $2 \cdot 6 = 12$ seating arrangements.

Answer: (B) 12. □

Problem 3.1.2 (2012 AMC 10B, Problem #11). A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

(A) 729 (B) 972 (C) 1024 (D) 2187 (E) 2304

Strategy. Friday's dessert is fixed, so start there and choose the desserts for the other days moving away from Friday. Then every other day has the same number of choices.

Solution. Friday's dessert is cake. Saturday's dessert must differ from Friday's, so it has 3 choices. Moving backward from Friday, Thursday's dessert must differ from Friday's (3 choices), Wednesday's must differ from Thursday's (3 choices), and in the same way Tuesday, Monday and Sunday each have 3 choices. Each of these six days has exactly 3 choices whatever was chosen before, so by the Multiplication Principle there are $3^6 = 729$ menus.

Choosing the days in calendar order does not work as directly. Thursday's dessert must differ from Wednesday's and must not be cake, since Friday's is cake. So Thursday has 3 choices if Wednesday's dessert is cake and 2 choices otherwise, and the count would need casework.

Answer: (A) 729. □

3.2. Adjacent Objects. To count arrangements in which certain objects must be next to each other, **glue** them into a single block. Arrange the block together with the other objects, then multiply by the number of ways to order the objects inside the block.

In general, if k particular objects among n distinct objects must form a block, the block and the other $n - k$ objects are $n - k + 1$ objects to arrange, and the block can be ordered in $k!$ ways. So there are $(n - k + 1)! \cdot k!$ arrangements.

Examples.

Problem 3.2.1. Six friends, including Ann and Ben, stand in a line for a photo. In how many orders can they stand if Ann and Ben must stand next to each other?

Strategy. Treat Ann and Ben as one block.

Solution. Glue Ann and Ben into one block. The block and the other 4 friends are 5 objects, which can be arranged in $5! = 120$ ways. Inside the block, Ann can be to the left or to the right of Ben, so the block can be ordered in $2! = 2$ ways. By the Multiplication Principle there are $120 \cdot 2 = 240$ orders.

Answer: 240. □

Problem 3.2.2 (2006 AMC 10A, Problem #18). A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?

(A) $10^4 \cdot 26^2$ (B) $10^3 \cdot 26^3$ (C) $5 \cdot 10^4 \cdot 26^2$ (D) $10^2 \cdot 26^4$ (E) $5 \cdot 10^3 \cdot 26^3$

Strategy. The two letters must be together, so glue them into one block. Choose where the block goes, then fill in the characters.

Solution. Glue the two letters into one block. A plate then consists of 5 units in a row: the block and the 4 digits. The block can be the first, second, third, fourth or fifth unit, so the letters occupy positions 1 and 2, 2 and 3, 3 and 4, 4 and 5, or 5 and 6 of the plate: 5 choices. Once these positions are chosen, each of the 2 letter positions holds any of 26 letters and each of the 4 digit positions holds any of 10 digits, since characters may repeat. By the Multiplication Principle there are

$$5 \cdot 26^2 \cdot 10^4$$

license plates.

There is no extra factor of 2 for the order inside the block: the 26^2 choices for the two letter positions already include both orders of two different letters.

Answer: (C) $5 \cdot 10^4 \cdot 26^2$. □

3.3. Non-Adjacent Objects. To count arrangements in which no two of certain objects are next to each other, first arrange the other objects. A row of m objects has $m + 1$ **gaps**: one before the first object, one between each two neighbors, and one after the last object. Then put the restricted objects into the gaps, at most one in each gap. Objects in different gaps are never next to each other, and every arrangement in which no two restricted objects are adjacent arises in exactly one way. If the k restricted objects are distinct, place them one at a time: the first has $m + 1$ gaps to choose from, the second has m , and so on, for a total of $P(m + 1, k)$ ways.

Examples.

Problem 3.3.1. Four boys and three girls stand in a row. In how many ways can they stand so that no two girls are next to each other?

Strategy. Arrange the boys first, then put the girls into the gaps between and around the boys.

Solution. The 4 boys can stand in $4! = 24$ orders. For each order they create 5 gaps:

_ B _ B _ B _ B _

No two girls are next to each other exactly when each gap holds at most one girl. The first girl can go into any of the 5 gaps, the second into any of the other 4, and the third into any of the remaining 3, so the girls can be placed in $5 \cdot 4 \cdot 3 = 60$ ways. By the Multiplication Principle there are $24 \cdot 60 = 1440$ arrangements.

Answer: 1440. □

Problem 3.3.2 (1994 AHSME, Problem #22). Nine chairs in a row are to be occupied by six students and Professors Alpha, Beta and Gamma. These three professors arrive before the six students and decide to choose their chairs so that each professor will be between two students. In how many ways can Professors Alpha, Beta and Gamma choose their chairs?

(A) 12 (B) 36 (C) 60 (D) 84 (E) 630

Strategy. Picture the six students in a row first. Each professor must sit in a gap between two students, and no two professors can share a gap.

Solution. Picture the final seating, and look at the six students in their row. They create 5 gaps between neighboring students, plus a gap at each end:

_ S _ S _ S _ S _ S _ S _

A professor in an end gap would not be between two students. Two professors in the same gap would sit next to each other, so one of them would not be between two students. Hence the professors sit in three different gaps among the 5 gaps between students. Conversely, any way of putting the three professors into three different gaps among these 5 gives a valid seating and determines the professors' chairs. For instance, professors in the first, third and fourth of these gaps sit in chairs 2, 5 and 7.

So the answer is the number of ways to put Alpha, Beta and Gamma into three different gaps among 5, which is $P(5, 3) = 5 \cdot 4 \cdot 3 = 60$. The order of the students does not matter, since the question asks only for the professors' chairs.

Answer: (C) 60. □

4. PERMUTATIONS OF A MULTISSET

Ordinary permutations are arrangements of distinct objects. Many problems involve objects that are not all distinct, such as the letters of a word with repeated letters, or objects that must appear in a given order relative to one another.

4.1. Repeated Objects.

Definition 4.1.1. A **multiset** is a collection of objects in which an object may appear more than once.

For example, the letters of the word MISSISSIPPI form a multiset with one M, four I's, four S's and two P's.

To count the distinct arrangements of a multiset, first treat all the objects as distinct by giving the identical ones labels, such as I_1, I_2, I_3, I_4 for the four I's. The n labeled objects then have $n!$ arrangements. This overcounts, because permuting the labels of identical objects among themselves does not produce a new arrangement once the labels are removed.

Theorem 4.1.2. Suppose a multiset of n objects consists of k_1 identical objects of one type, k_2 identical objects of a second type, and so on up to k_r identical objects of an r th type, where $k_1 + k_2 + \cdots + k_r = n$. Then the number of distinct arrangements of the multiset in a row is

$$\frac{n!}{k_1! k_2! \cdots k_r!}.$$

Proof. Label the objects of each type so that all n objects are distinct. Take any arrangement of the multiset. The k_1 labels of the first type can be given to the k_1 positions holding that type in $k_1!$ ways, the labels of the second type in $k_2!$ ways, and so on. By the Multiplication Principle, each arrangement of the multiset can be labeled in exactly $k_1! k_2! \cdots k_r!$ ways. Every one of the $n!$ arrangements of the labeled objects arises in this way from exactly one arrangement of the multiset, the one obtained by removing its labels. Therefore

$$(\text{number of arrangements of the multiset}) \cdot k_1! k_2! \cdots k_r! = n!,$$

which gives the formula. □

The formula divides $n!$, the number of arrangements of n distinct objects, by the number of ways to permute the identical objects of each type among themselves. A type that appears only once contributes $1! = 1$, so only repeated objects change the count. For MISSISSIPPI, with $n = 11$,

$$\frac{11!}{1! 4! 4! 2!} = \frac{39,916,800}{1152} = 34,650.$$

Example.

Problem 4.1.3 (1985 AHSME, Problem #11). How many distinguishable rearrangements of the letters in CONTEST have both the vowels first? (For instance, OETCNST is one such arrangement, but OTETSNC is not.)

(A) 60 (B) 120 (C) 240 (D) 720 (E) 2520

Strategy. The vowels are restricted to the first two positions, so place them first. Then arrange the consonants, which include two T's.

Solution. The letters of CONTEST are the vowels O and E and the consonants C, N, T, S, T. The vowels fill the first two positions in $2! = 2$ orders, OE or EO. The consonants fill the last five positions. They form a multiset of five letters in which T appears twice, so they have

$$\frac{5!}{2!} = \frac{120}{2} = 60$$

distinct arrangements. By the Multiplication Principle there are $2 \cdot 60 = 120$ rearrangements.

Answer: (B) 120. □

4.2. Relative Order. Some problems require certain objects to appear in a fixed order relative to one another, though not necessarily next to each other. For example, an arrangement of the letters A, B, C, D, E may be required to have A somewhere before C.

Theorem 4.2.1. *If k particular objects among n distinct objects must appear in a given relative order, the number of arrangements of the n objects in a row is*

$$\frac{n!}{k!}.$$

Proof. Without the condition there are $n!$ arrangements. In each arrangement the k particular objects occupy some k positions. Put two arrangements in the same group when they differ only in the order of the k particular objects within those positions. Each group contains $k!$ arrangements, one for each order of the k objects, and exactly one of them has the required order. So the number of valid arrangements is the number of groups, $n!/k!$. $\square$

The same count comes from treating the k particular objects as identical placeholders. Arrange the n objects with the k particular ones replaced by copies of one symbol, which can be done in $n!/k!$ ways. Then fill the placeholders from left to right with the particular objects in the required order. This method also works when there are several groups of objects, each with its own required order, and when some of the objects are already identical.

Examples.

Problem 4.2.2. How many distinct arrangements of the letters in the word LETTERS are there such that the two E's appear before both T's?

Strategy. Only the relative order of the four letters E, E, T, T matters. Either divide the total by the number of possible orders of these four letters, or replace them by placeholders.

Solution. The word LETTERS has seven letters: one L, two E's, two T's, one R and one S.

Method 1: Symmetry. Without the condition, the number of distinct arrangements is

$$\frac{7!}{2!2!} = \frac{5040}{4} = 1260.$$

The letters E, E, T, T can appear in $4!/(2!2!) = 6$ relative orders:

$$\text{EETT, ETET, ETTE, TEET, TETE, TTEE.}$$

Put two arrangements of LETTERS in the same group when they differ only in the order of E, E, T, T within the four positions these letters occupy. Each group contains one arrangement for each of the six orders, and only the order EETT has both E's before both T's. So the number of valid arrangements is $1260/6 = 210$.

Method 2: Placeholders. Replace each of E, E, T, T by the symbol X, and arrange the letters L, R, S, X, X, X, X. The number of distinct arrangements is

$$\frac{7!}{4!} = \frac{5040}{24} = 210.$$

Replacing the four X's from left to right by E, E, T, T turns each of these into a valid arrangement of LETTERS, and every valid arrangement arises from exactly one of them.

Answer: 210. $\square$

Problem 4.2.3 (2001 AMC 12, Problem #16). A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

- (A) $8!$ (B) $2^8 8!$ (C) $(8!)^2$ (D) $\frac{16!}{2^8}$ (E) $16!$

Strategy. The 16 items are distinct, and on each leg the sock and the shoe must appear in a fixed relative order. Use a pair of placeholders for each leg.

Solution. For each leg, replace its sock and its shoe by two copies of the leg's number. The 16 items become a multiset with two 1's, two 2's, and so on up to two 8's, which has

$$\frac{16!}{2!2!\cdots 2!} = \frac{16!}{2^8}$$

distinct arrangements, with eight factors of $2!$ in the denominator. Each arrangement gives exactly one valid order: for each leg, the first copy of the leg's number is the sock and the second copy is the shoe. Every valid order arises in this way from exactly one arrangement, so there are $16!/2^8$ orders.

Equivalently, of the $16!$ orders of the items, the condition on each leg keeps one of the two relative orders of that leg's sock and shoe, so the count is $16!$ divided by 2 once for each leg.

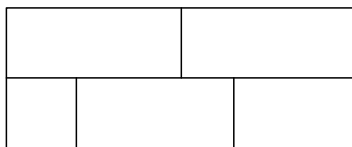
Answer: (D) $\frac{16!}{2^8}$.

□

5. PROBLEMS

Beginner.

- (2020 AMC 10B, Problem #5) How many distinguishable arrangements are there of 1 brown tile, 1 purple tile, 2 green tiles, and 3 yellow tiles in a row from left to right? (Tiles of the same color are indistinguishable.)
(A) 210 (B) 420 (C) 630 (D) 840 (E) 1050
- (2003 AMC 10B, Problem #10) Nebraska, the home of the AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by four digits. Each new license plate consists of three letters followed by three digits. By how many times is the number of possible license plates increased?
(A) $\frac{26}{10}$ (B) $\frac{26^2}{10^2}$ (C) $\frac{26^2}{10}$ (D) $\frac{26^3}{10^3}$ (E) $\frac{26^3}{10^2}$
- (2009 AMC 10B, Problem #11) How many 7-digit palindromes (numbers that read the same backward as forward) can be formed using the digits 2, 2, 3, 3, 5, 5, 5?
(A) 6 (B) 12 (C) 24 (D) 36 (E) 48
- (2014 AMC 10A, Problem #4) Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6
- (2022 AMC 10A, Problem #9) A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color—red, orange, yellow, blue, or green—so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?



- (A) 120 (B) 270 (C) 360 (D) 540 (E) 720

Intermediate.

- (2011 AMC 10A, Problem #13) How many even integers are there between 200 and 700 whose digits are all different and come from the set $\{1, 2, 5, 7, 8, 9\}$?
(A) 12 (B) 20 (C) 72 (D) 120 (E) 200
- (1988 AHSME, Problem #18) At the end of a professional bowling tournament, the top 5 bowlers have a play-off. First #5 bowls #4. The loser receives 5th prize and the winner bowls #3 in another game. The loser of this game receives 4th prize and the winner bowls #2. The loser of this game receives 3rd prize and the winner bowls #1. The winner of this game gets 1st prize and the loser gets 2nd prize. In how many orders can bowlers #1 through #5 receive the prizes?
(A) 10 (B) 16 (C) 24 (D) 120 (E) none of these

8. (2020 AMC 10A, Problem #18) Let (a, b, c, d) be an ordered quadruple of not necessarily distinct integers, each one of them in the set $\{0, 1, 2, 3\}$. For how many such quadruples is it true that $a \cdot d - b \cdot c$ is odd? (For example, $(0, 3, 1, 1)$ is one such quadruple, because $0 \cdot 1 - 3 \cdot 1 = -3$ is odd.)
(A) 48 (B) 64 (C) 96 (D) 128 (E) 192
9. (2005 AMC 10B, Problem #20) What is the average (mean) of all 5-digit numbers that can be formed by using each of the digits 1, 3, 5, 7, and 8 exactly once?
(A) 48000 (B) 49999.5 (C) 53332.8 (D) 55555 (E) 56432.8
10. (2022 AMC 12A, Problem #10) What is the number of ways the numbers from 1 to 14 can be split into 7 pairs such that for each pair, the greater number is at least 2 times the smaller number?
(A) 108 (B) 120 (C) 126 (D) 132 (E) 144

Advanced.

11. (1982 AHSME, Problem #11) How many integers with four different digits are there between 1,000 and 9,999 such that the absolute value of the difference between the first digit and the last digit is 2?
(A) 672 (B) 784 (C) 840 (D) 896 (E) 1,008
12. (2017 AMC 10A, Problem #19) Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?
(A) 12 (B) 16 (C) 28 (D) 32 (E) 40
13. (2024 AMC 10B, Problem #17) In a race among 5 snails, there is at most one tie, but that tie can involve any number of snails. For example, the result of the race might be that Dazzler is first; Abby, Cyrus, and Elroy are tied for second; and Bruna is fifth. How many different results of the race are possible?
(A) 180 (B) 361 (C) 420 (D) 431 (E) 720
14. (2018 AMC 10B, Problem #18) Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?
(A) 60 (B) 72 (C) 92 (D) 96 (E) 120