COUNTING PRINCIPLES AND PERMUTATIONS FOR AMC 10: PRACTICE SET

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1. Problems

Beginner.

1. The six letters of the word COFFEE are arranged in a row. In how many distinct arrangements are the two F's next to each other?

2. (2022 AMC 10B, Problem #9) The sum

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{2021}{2022!}$$

can be expressed as $a-\frac{1}{b!}$, where a and b are positive integers. What is $a+b$?

(A) 2020 **(B)** 2021 **(C)** 2022 **(D)** 2023 **(E)** 2024

3. (2003 AMC 10B, Problem #13) Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8)=8$ and $\clubsuit(123)=1+2+3=6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x))=3$?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 9 **(E)** 10

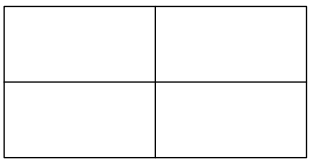
4. (2003 AMC 10B, Problem #16) A restaurant offers three desserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a dessert. What is the least number of main courses that the restaurant should offer so that a customer could have a different dinner each night in the year 2003?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

5. How many distinct arrangements of the letters in the word PARADE have no two vowels next to each other?

Intermediate.

6. (2021 Fall AMC 10A, Problem #18) A farmer's rectangular field is partitioned into a 2 by 2 grid of 4 rectangular sections as shown in the figure. In each section the farmer will plant one crop: corn, wheat, soybeans, or potatoes. The farmer does not want to grow corn and wheat in any two sections that share a border, and the farmer does not want to grow soybeans and potatoes in any two sections that share a border. Given these restrictions, in how many ways can the farmer choose crops to plant in each of the four sections of the field?



(A) 12 **(B)** 64 **(C)** 84 **(D)** 90 **(E)** 144

7. (2019 AMC 12A, Problem #13) How many ways are there to paint each of the integers 2, 3, ..., 9 either red, green, or blue so that each number has a different color from each of its proper divisors?

(A) 144 **(B)** 216 **(C)** 256 **(D)** 384 **(E)** 432

8. (2003 AIME I, Problem #1) Given that $\frac{((3!)!)!}{3!}=k\cdot n!$, where k and n are positive integers and n is as large as possible, find $k+n$.

9. (2003 AIME II, Problem #3) Define a *good word* as a sequence of letters that consists only of the letters A, B, and C — some of these letters may not appear in the sequence — and in which A is never immediately followed by B, B is never immediately followed by C, and C is never immediately followed by A. How many seven-letter good words are there?

10. A chef prepares seven different dishes, one at a time. Among them are bread, soup, stew, salad and dessert. The bread must be prepared before both the soup and the stew, and the salad must be prepared before the dessert. In how many orders can the chef prepare the seven dishes?

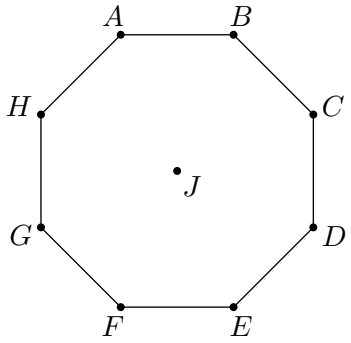
Advanced.

11. (2008 AMC 10B, Problem #21) Ten chairs are evenly spaced around a round table and numbered clockwise from 1 through 10. Five married couples are to sit in the chairs with men and women alternating, and no one is to sit either next to or directly across from his or her spouse. How many seating arrangements are possible?

(A) 240 **(B)** 360 **(C)** 480 **(D)** 540 **(E)** 720

12. (2007 AIME II, Problem #1) A mathematical organization is producing a set of commemorative license plates. Each plate contains a sequence of five characters chosen from the four letters in AIME and the four digits in 2007. No character may appear in a sequence more times than it appears among the four letters in AIME or the four digits in 2007. A set of plates in which each possible sequence appears exactly once contains N license plates. Find $\frac{N}{10}$.

13. (2013 AMC 10B, Problem #22) The regular octagon $ABCDEFGH$ has its center at J. Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE, BJF, CJG, and DJH are all equal. In how many ways can this be done?



(A) 384 **(B)** 576 **(C)** 1152 **(D)** 1680 **(E)** 3456

14. (2005 AIME I, Problem #5) Robert has 4 indistinguishable gold coins and 4 indistinguishable silver coins. Each coin has an engraving of a face on one side, but not on the other. He wants to stack the eight coins on a table into a single stack so that no two adjacent coins are face to face. Find the number of possible distinguishable arrangements of the 8 coins.

15. (2013 AMC 10A, Problem #24) Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

(A) 540 **(B)** 600 **(C)** 720 **(D)** 810 **(E)** 900

2. Answer Key

Problem	Letter	Value
1		60
2	D	2023
3	E	10
4	E	8
5		72
6	C	84
7	E	432
8		839
9		192
10		840
11	C	480
12		372
13	C	1152
14		630
15	E	900