POLYNOMIALS, INEQUALITIES, AND NUMBER THEORY FOR AMC 10: PRACTICE SET

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1. Problems

Beginner.

1. (2022 AMC 10B, Problem #4) A donkey suffers an attack of hiccups and the first hiccup happens at 4:00 one afternoon. Suppose that the donkey hiccups regularly every 5 seconds. At what time does the donkey's 700th hiccup occur?

(A) 15 seconds after 4:58 **(B)** 20 seconds after 4:58 **(C)** 25 seconds after 4:58 **(D)** 30 seconds after 4:58 **(E)** 35 seconds after 4:58

2. (2018 AMC 10A, Problem #7) For how many (not necessarily positive) integer values of n is the following value an integer?

$$4000 \cdot \left(\frac{2}{5}\right)^n$$

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

3. (2021 AMC 10A, Problem #10) Which of the following is equivalent to

$$(2+3)(2^2+3^2)(2^4+3^4)(2^8+3^8)(2^{16}+3^{16})(2^{32}+3^{32})(2^{64}+3^{64})?$$

(A) $3^{127}+2^{127}$ **(B)** $3^{127}+2^{127}+2\cdot 3^{63}+3\cdot 2^{63}$ **(C)** $3^{128}-2^{128}$ **(D)** $3^{128}+2^{128}$ **(E)** 5^{127}

4. (2013 AMC 10B, Problem #9) Three positive integers are each greater than 1, have a product of 27000, and are pairwise relatively prime. What is their sum?

(A) 100 **(B)** 137 **(C)** 156 **(D)** 160 **(E)** 165

5. (1983 AHSME, Problem #14) The units digit of $3^{1001}7^{1002}13^{1003}$ is

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

Intermediate.

6. (2017 AMC 10B, Problem #14) An integer N is selected at random in the range $1 \leq N \leq 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1?

(A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{5}$ **(E)** 1

7. (2003 AMC 10B, Problem #25) How many distinct four-digit numbers are divisible by 3 and have 23 as their last two digits?

(A) 27 **(B)** 30 **(C)** 33 **(D)** 81 **(E)** 90

8. (2009 AMC 10B, Problem #14) On Monday, Millie puts a quart of seeds, 25% of which are millet, into a bird feeder. On each successive day she adds another quart of the same mix of seeds without removing any seeds that are left. Each day the birds eat only 25% of the millet in the feeder, but they eat all of the other seeds. On which day, just after Millie has placed the seeds, will the birds find that more than half the seeds in the feeder are millet?

(A) Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday **(E)** Saturday

9. (2003 AMC 12B, Problem #18) Let x and y be positive integers such that $7x^5 = 11y^{13}$. The minimum possible value of x has a prime factorization $a^c b^d$. What is $a+b+c+d$?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

10. (1958 AHSME, Problem #46) For values of x less than 1 but greater than -4, the expression

$$\frac{x^2 - 2x + 2}{2x - 2}$$

has:

(A) no maximum or minimum value **(B)** a minimum value of 1
(C) a maximum value of 1 **(D)** a minimum value of -1
(E) a maximum value of -1

Advanced.

11. (2016 AMC 10B, Problem #17) All the numbers 2, 3, 4, 5, 6, 7 are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

(A) 312 **(B)** 343 **(C)** 625 **(D)** 729 **(E)** 1680

12. (2024 AMC 12B, Problem #17) Integers a and b are randomly chosen without replacement from the set of integers with absolute value not exceeding 10. What is the probability that the polynomial $x^3 + ax^2 + bx + 6$ has 3 distinct integer roots?

(A) $\frac{1}{240}$ **(B)** $\frac{1}{221}$ **(C)** $\frac{1}{105}$ **(D)** $\frac{1}{84}$ **(E)** $\frac{1}{63}$

13. (2015 AIME I, Problem #3) There is a prime number p such that $16p + 1$ is the cube of a positive integer. Find p.

14. (2016 AMC 10A, Problem #22) For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

(A) 110 **(B)** 191 **(C)** 261 **(D)** 325 **(E)** 425

15. (1977 AHSME, Problem #28) Let $g(x) = x^5 + x^4 + x^3 + x^2 + x + 1$. What is the remainder when the polynomial $g(x^{12})$ is divided by the polynomial $g(x)$?

(A) 6 **(B)** $5 - x$ **(C)** $4 - x + x^2$ **(D)** $3 - x + x^2 - x^3$
(E) $2 - x + x^2 - x^3 + x^4$

2. Answer Key

Problem	Letter	Value
1	A	15 seconds after 4:58
2	E	9
3	C	$3^{128} - 2^{128}$
4	D	160
5	E	9
6	D	$\frac{4}{5}$
7	B	30
8	D	Friday
9	B	31
10	E	a maximum value of -1
11	D	729
12	C	$\frac{1}{105}$
13		307
14	D	325
15	A	6