QUADRATICS, THE DISCRIMINANT, AND VIETA'S FORMULAS FOR AMC 10: PRACTICE SET

ANDY JO

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1. Problems

Beginner.

1. (2019 AMC 10A, Problem #3) Ana and Bonita were born on the same date in different years, n years apart. Last year Ana was 5 times as old as Bonita. This year Ana's age is the square of Bonita's age. What is n?

(A) 3 **(B)** 5 **(C)** 9 **(D)** 12 **(E)** 15

2. (1957 AHSME, Problem #36) If $x + y = 1$, then the largest value of xy is:

(A) 1 **(B)** 0.5 **(C)** an irrational number about 0.4 **(D)** 0.25 **(E)** 0

3. (1966 AHSME, Problem #23) If x is real and $4y^2 + 4xy + x + 6 = 0$, then the complete set of values of x for which y is real, is:

(A) $x \leq -2$ or $x \geq 3$ **(B)** $x \leq 2$ or $x \geq 3$ **(C)** $x \leq -3$ or $x \geq 2$
(D) $-3 \leq x \leq 2$ **(E)** $-2 \leq x \leq 3$

4. (2016 AMC 10B, Problem #9) All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 16

5. (2002 AMC 10B, Problem #10) Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b. What is the pair (a, b)?

(A) $(-2, 1)$ **(B)** $(-1, 2)$ **(C)** $(1, -2)$ **(D)** $(2, -1)$ **(E)** $(4, 4)$

Intermediate.

6. (2020 AMC 10B, Problem #9) How many ordered pairs of integers (x, y) satisfy the equation

$$x^{2020} + y^2 = 2y?$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many

7. (1960 AHSME, Problem #11) For a given value of k, the product of the roots of

$$x^2 - 3kx + 2k^2 - 1 = 0$$

is 7. The roots may be characterized as:

(A) integral and positive **(B)** integral and negative **(C)** rational, but not integral
(D) irrational **(E)** imaginary

8. (1959 AHSME, Problem #38) If $4x + \sqrt{2x} = 1$, then x:

(A) is an integer **(B)** is fractional **(C)** is irrational **(D)** is imaginary
(E) may have two different values

9. (2017 AMC 12B, Problem #8) The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

(A) $\frac{\sqrt{3}-1}{2}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\sqrt{5}-1}{2}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{6}-1}{2}$

10. (1964 AHSME, Problem #30) If

$$(7+4\sqrt{3})x^2+(2+\sqrt{3})x-2=0,$$

the larger root minus the smaller root is:

(A) $-2+3\sqrt{3}$ **(B)** $2-\sqrt{3}$ **(C)** $6+3\sqrt{3}$ **(D)** $6-3\sqrt{3}$ **(E)** $3\sqrt{3}+2$

Advanced.

11. (2019 AMC 10A, Problem #19) What is the least possible value of

$$(x+1)(x+2)(x+3)(x+4)+2019,$$

where x is a real number?

(A) 2017 **(B)** 2018 **(C)** 2019 **(D)** 2020 **(E)** 2021

12. (1991 AIME, Problem #1) Find x^2+y^2 if x and y are positive integers such that

$$xy+x+y=71,$$
$$x^2y+xy^2=880.$$

13. (2015 AMC 10A, Problem #23) The zeros of the function

$$f(x)=x^2-ax+2a$$

are integers. What is the sum of the possible values of a?

(A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

14. (1985 AHSME, Problem #19) Consider the graphs of $y=Ax^2$ and $y^2+3=x^2+4y$, where A is a positive constant and x and y are real variables. In how many points do the two graphs intersect?

(A) exactly 4 **(B)** exactly 2 **(C)** at least 1, but the number varies for different positive values of A **(D)** 0 for at least one positive value of A **(E)** none of these

15. (2010 AMC 12B, Problem #23) Monic quadratic polynomials $P(x)$ and $Q(x)$ have the property that $P(Q(x))$ has zeros at $x=-23$, -21, -17, and -15, and $Q(P(x))$ has zeros at $x=-59$, -57, -51, and -49. What is the sum of the minimum values of $P(x)$ and $Q(x)$?

(A) -100 **(B)** -82 **(C)** -73 **(D)** -64 **(E)** 0

2. Answer Key

Problem	Letter	Value
1	D	12
2	D	0.25
3	A	$x \leq -2$ or $x \geq 3$
4	C	8
5	C	$(1, -2)$
6	D	4
7	D	irrational
8	B	is fractional
9	C	$(\sqrt{5}-1)/2$
10	D	$6 - 3\sqrt{3}$
11	B	2018
12		146
13	C	16
14	A	exactly 4
15	A	-100