SYSTEMS, RATES, AND WORK PROBLEMS FOR AMC 10: PRACTICE SET

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Contents

1. Problems 2
2. Answer Key 5

1. Problems

Beginner.

1. (2021 AMC 10B, Problem #3) In an after-school program for juniors and seniors, there is a debate team with an equal number of students from each class on the team. Among the 28 students in the program, 25% of the juniors and 10% of the seniors are on the debate team. How many juniors are in the program?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 11 **(E)** 20

2. (2022 AMC 10A, Problem #3) The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the absolute value of the difference between the first and second numbers?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. (2021 Fall AMC 10A, Problem #4) Mr. Lopez has a choice of two routes to get to work. Route A is 6 miles long, and his average speed along this route is 30 miles per hour. Route B is 5 miles long, and his average speed along this route is 40 miles per hour, except for a $\frac{1}{2}$-mile stretch in a school zone where his average speed is 20 miles per hour. By how many minutes is Route B quicker than Route A?

(A) $2\frac{3}{4}$ **(B)** $3\frac{3}{4}$ **(C)** $4\frac{1}{2}$ **(D)** $5\frac{1}{2}$ **(E)** $6\frac{3}{4}$

4. (2019 AMC 10B, Problem #3) In a high school with 500 students, 40% of the seniors play a musical instrument, while 30% of the non-seniors do not play a musical instrument. In all, 46.8% of the students do not play a musical instrument. How many non-seniors play a musical instrument?

(A) 66 **(B)** 154 **(C)** 186 **(D)** 220 **(E)** 266

5. (2017 AMC 10A, Problem #4) Mia is "helping" her mom pick up 30 toys that are strewn on the floor. Mia's mom manages to put 3 toys into the toy box every 30 seconds, but each time immediately after those 30 seconds have elapsed, Mia takes 2 toys out of the box. How much time, in minutes, will it take Mia and her mom to put all 30 toys into the box for the first time?

(A) 13.5 **(B)** 14 **(C)** 14.5 **(D)** 15 **(E)** 15.5

Intermediate.

6. (1959 AHSME, Problem #30) A can run around a circular track in 40 seconds. B, running in the opposite direction, meets A every 15 seconds. What is B's time to run around the track, expressed in seconds?

(A) $12\frac{1}{2}$ **(B)** 24 **(C)** 25 **(D)** $27\frac{1}{2}$ **(E)** 55

7. (2006 AMC 10B, Problem #13) Joe and JoAnn each bought 12 ounces of coffee in a 16-ounce cup. Joe drank 2 ounces of his coffee and then added 2 ounces of cream. JoAnn added 2 ounces of cream, stirred the coffee well, and then drank 2 ounces. What is the resulting ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee?

(A) $\frac{6}{7}$ **(B)** $\frac{13}{14}$ **(C)** 1 **(D)** $\frac{14}{13}$ **(E)** $\frac{7}{6}$

8. (1958 AHSME, Problem #24) A man travels m feet due north at 2 minutes per mile. He returns due south to his starting point at 2 miles per minute. The average rate in miles per hour for the entire trip is:
(A) 75 **(B)** 48 **(C)** 45 **(D)** 24
(E) impossible to determine without knowing the value of m

9. (1956 AHSME, Problem #47) An engineer said he could finish a highway section in 3 days with his present supply of a certain type of machine. However, with 3 more of these machines the job could be done in 2 days. If the machines all work at the same rate, how many days would it take to do the job with one machine?
(A) 6 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 36

10. (1973 AHSME, Problem #24) The check for a luncheon of 3 sandwiches, 7 cups of coffee and one piece of pie came to \$3.15. The check for a luncheon consisting of 4 sandwiches, 10 cups of coffee and one piece of pie came to \$4.20 at the same place. The cost of a luncheon consisting of one sandwich, one cup of coffee and one piece of pie at the same place will come to
(A) \$1.70 **(B)** \$1.65 **(C)** \$1.20 **(D)** \$1.05 **(E)** \$0.95

Advanced.

11. (1986 AIME, Problem #4) Determine $3x_4 + 2x_5$ if x_1, x_2, x_3, x_4, and x_5 satisfy the system

$$\begin{aligned} 2x_1 + x_2 + x_3 + x_4 + x_5 &= 6, \\ x_1 + 2x_2 + x_3 + x_4 + x_5 &= 12, \\ x_1 + x_2 + 2x_3 + x_4 + x_5 &= 24, \\ x_1 + x_2 + x_3 + 2x_4 + x_5 &= 48, \\ x_1 + x_2 + x_3 + x_4 + 2x_5 &= 96. \end{aligned}$$

12. (1973 AHSME, Problem #10) If n is a real number, then the simultaneous system

$$\begin{cases} nx + y = 1, \\ ny + z = 1, \\ x + nz = 1 \end{cases}$$

has no solution if and only if n is equal to
(A) -1 **(B)** 0 **(C)** 1 **(D)** 0 or 1 **(E)** $\frac{1}{2}$

13. (1955 AHSME, Problem #50) In order to pass B going 40 mph on a two-lane highway A, going 50 mph, must gain 30 feet. Meantime, C, 210 feet from A, is headed toward him at 50 mph. If B and C maintain their speeds, then, in order to pass safely, A must increase his speed by:
(A) 30 mph **(B)** 10 mph **(C)** 5 mph **(D)** 15 mph **(E)** 3 mph

14. (2011 AIME I, Problem #1) Jar A contains four liters of a solution that is 45% acid. Jar B contains five liters of a solution that is 48% acid. Jar C contains one liter of a solution that is k% acid. From jar C, $\frac{m}{n}$ liters of the solution is added to jar A, and the remainder of the solution in jar C is added to jar B. At the end both jar A and jar B contain solutions that are 50% acid. Given that m and n are relatively prime positive integers, find $k + m + n$.

15. (2012 AMC 10A, Problem #19) Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8:00 AM, and all three always take the same amount of time to eat lunch.

On Monday the three of them painted 50% of a house, quitting at 4:00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2:12 PM. On Wednesday Paula worked by herself and finished the house by working until 7:12 P.M.

How long, in minutes, was each day's lunch break?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 60

2. Answer Key

Problem	Letter	Value
1	C	8
2	E	5
3	B	$3\frac{3}{4}$
4	B	154
5	B	14
6	B	24
7	E	$\frac{7}{6}$
8	B	48
9	D	18
10	D	\$1.05
11		181
12	A	-1
13	C	5 mph
14		85
15	D	48